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Analysing the precision of resource aware localisation algorithms for wireless sensor networks

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Abstract — **One of the key issues in wireless sensor networks is the precise determination of positions of arbitrarily located sensor nodes with low complexity and lowest energy consumption. Depending on the application, different accuracy levels are required. It can be distinguished between coarse-grained algorithms with lower hardware requirements, but low precision and fine-grained algorithms with high accuracy, but which are also very resource-intensive. In this paper we investigate and compare the precision of selected methods with respect to the network geometry and highly inaccurate distance measurements.**

Keywords: wireless sensor networks, geosensors, localisation, accuracy, precise positioning

I. INTRODUCTION

A wireless sensor network (WSN) consists of hundreds or thousands of small wireless devices, able to sense the environment, compute simple tasks and to communicate with each other. These nodes consist of sensors, actuators, a low power processor, small memory, and a communication module. Thousands of these nodes form a large wireless sensor network to monitor huge inaccessible terrains (Akyildiz et al., 2002). Due to the desired node size of a few cubic millimeters, the dimensions of the communication module and the battery are critical. Consequently, the scarcest resource within a network is the available energy. Power-optimised hardware and energy-aware algorithms must therefore be developed.

Gathered information is only useful if linked to a geographical position. A possible approach is to equip all sensor nodes with a commonly used localisation techniques such as the "Global System for Mobile Communication" (GSM) or "Global Navigation Satellite System" (GNSS). These systems are however, due to the size of the equipment and the high energy requirements, unsuitable for miniaturised sensor nodes and can therefore only be used for a small number of nodes, further called beacons. To localise the remaining nodes several approaches exist in literature. In this paper we investigate and compare error sources and the resulting positional accuracy of selected localisation algorithms. The outline of this paper is as follows: Section II gives an overview of localisation in wireless sensor networks. In Section III we describe the chosen algorithms in more detail. The simulation setup and results are presented in Section IV before we conclude the paper in Section V.

II. LOCALISATION IN WIRELESS SENSOR NETWORKS

Localisation algorithms are mainly classified into fine-grained and coarse-grained, depending on the complexity, redundancy and achieved accuracy. Most of the coarse-grained localisation algorithms need only few resources, but estimate a position with a relatively high positional error. Different coarse-grained approaches exist in the literature. For example, He et al. completely avoid distances in their approach (He et al. 2003). In another approach by Bulusu et al. every sensor node calculates the centroid as its own position (Bulusu, 2000). This approach was extended with distances as weights, which further improved the precision (Blumenthal et al. 2005). Finally, a very simple method is to take the position of the nearest beacon position as position estimate.

In contrast, exact (or fine-grained) localisation of a sensor node features high precision and is based on solving either (i) linear systems with only the minimum required number of beacons or (ii) a high amount of beacons and distances to them, which makes use of the high redundancy in the network and leads to optimisation problems. In more detail, with at least three beacons (in two-dimensions), sensor nodes estimate their positions via trilateration. More beacons than required result in an over-determined system of equations that must be solved with e.g. a least squares method (multilateration). Although a multilateration produces precise results, it is complex and resource-intensive and without optimisation strategies not feasible on resource-limited sensor nodes.

III. LOCALISATION ALGORITHMS

This section describes the analysed localisation algorithms in this paper. Here, we assume n as the number of beacons with known positions.

A. *Weighted Centroid Localization (WCL)*

"Weighted Centroid Localization" (WCL), as a coarse-grained localisation algorithm, assumes randomly deployed beacons. In the first phase all beacons broadcast their position $B_j(x,y)$ to all sensor nodes within their transmission range. While receiving the data, every sensor node estimates the distance and stores it together with the corresponding beacon position. After all positions are gathered, the sensor node calculates its approximate position by a weighted centroid determination with all n beacons in transmission range:

$$P(x,y)=\frac{\sum_{j=1}^{n}\left(w_j\cdot B_j(x,y)\right)}{\sum_{j=1}^{n}w_j} \tag{1}$$

WCL uses distance information as weights w_j. Shorter distances lead to higher weights than longer ones. Further, every coordinate of a beacons position obtains a weight depending on the distance.

B. *Fine-grained Localisation algorithm (FGL)*

One of the most cited localisation methods is the "Fine Grained Localization" by Savvides et al. (Savvides, 2001). This algorithm requires at minimum three beacons in the Euclidean space. The calculation is based on the Least Squares Method with an over-determined system of equations. The procedure can be described as follows: all sensor nodes measure the distances to all available beacons in transmission range. With the Euclidean distance d_j, where P(x,y) is the estimated position of the sensor node and $B_j(x,y)$ is the exact position of the j-th beacon this results in the over-determined system of equations:

$$d_j^2=\left(x-x_j\right)^2+\left(y-y_j\right)^2=b_j \quad j=1\ldots n \tag{2}$$

This non-linear system of equations will be solved iteratively using start values and applying Taylor series for linearisation. This can be written in the form A**x**=**b** and solved by the Least Squares (LS) method that leads to $\mathbf{x}=(A^TA)^{-1}A^T\mathbf{b}$. Finally, the position vector can be solved on every sensor node completely distributed. In practice, starting values are not available. It is conceivable that energy aware coarse-grained algorithms may be applied to generate start values (Born et al. 2006).

C. *Distributed Least Squares algorithm (DLS)*

The "Distributed Least Squares" localisation algorithm (DLS) is also based on the Least Squares Method and starts with the creation of an over-determined system of non-linear Euclidean distances of the form (2). But in contrast to FGL, this system of equations is converted with a term extension (Reichenbach et al., 2006):

$$\left(x-x_j\right)\cdot\left(x_i-x_j\right)+\left(y-y_i\right)\cdot\left(y_i-y_j\right)=\frac{1}{2}\left(r_j^2-r_i^2+d_{ij}^2\right)= \tag{3}$$

This leads to the form A**x**=**b**, where A is the coefficient matrix, **b** is the right side vector and **x** is the solution vector. By applying the Least Squares Method we obtain the known normal equation system $\mathbf{x}=(A^TA)^{-1}A^T\mathbf{b}$. The matrices in this equation have two important benefits. First, all elements in the coefficient matrix **A** are generated by beacon positions $B_j(x,y)$ only. By assuming that we can establish communication links between sensor nodes and all beacons, matrix A is the same on every sensor node. Second, the vector **b** contains the distances between the sensor node and beacons r_j that must be estimated on every sensor node independently. Given these facts, the normal equation can be split into two parts - a more complex part, the pre-calculation: $A_p=(A^TA)^{-1}A^T$ and a simple part: $A_p\mathbf{b}$, which is further called the post-calculation. Here, the pre-calculation is executed on one powerful base station, which additionally avoids high redundancy, because the pre-calculation is normally executed on all sensor nodes. Only the simple post-calculation has to be executed on every sensor node with its individual distance measurements.

D. *Resource Aware Localization algorithm (RAL)*

In principal, RAL follows the same algorithm as DLS. The difference here is that the conversion of the non-linear system of equations will be carried out by resolving and substituting the system of equations (Born et al., in press):

$$d_i^2-x_i^2-y_i^2=u-2xx_i-2yy_i=b_j \tag{4}$$

with $u=x^2+y^2$

The difference between RAL and DLS is marginal but has an impact in the positional accuracy if erroneous distances are assumed. Furthermore, RAL requires less computation effort which helps to conserve resources on the energy-limited sensor nodes.

IV. SIMULATION AND RESULTS

In our simulations we examined the accuracy achieved by the algorithms introduced in the last section. As the localisation algorithms do not support angles in their current state, only distance measurements have been used. The exact Euclidean distances between the beacons and sensor nodes have therefore been distorted within the given confidence interval. The simulations have been carried out empirically, meaning that systematic as well as random errors have been considered.

A. *Simulation setup*

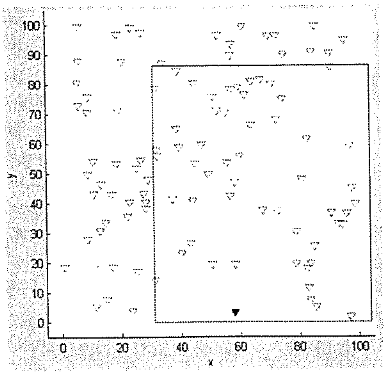

Figure 1. Test field

For the simulations Matlab is used. The simulation setup for the first investigation is as follows (Fig. 1): The test field has dimensions of 100m x 100m with 100 beacons, randomly distributed. It is assumed that the sensor node is within communication range of all beacons, whereas the distances have been distorted by a variance of 10%, following a Gaussian distribution. The required power for the weights in WCL is 2. Fig. 2 shows in detail the different point error of these localisation algorithms under same conditions.

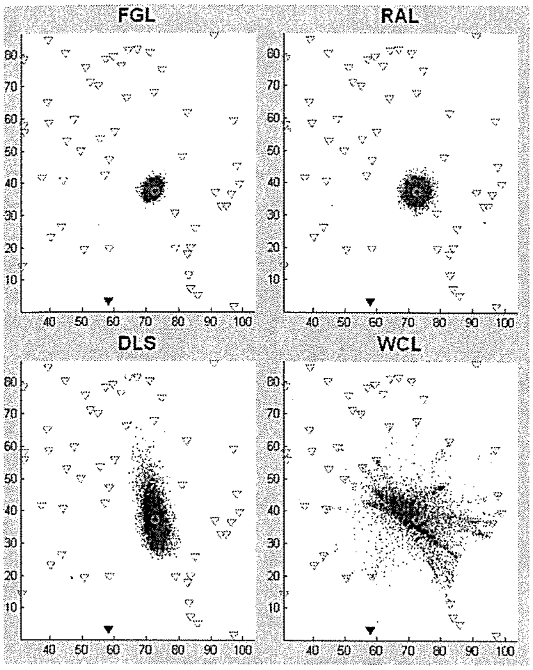

Figure 2. Point error (detail) random beacon deployment

For the second simulation we examined the systematic influence on all algorithms. For this, 100 beacons have been distributed circularly with a radius of 50m (Fig. 3).

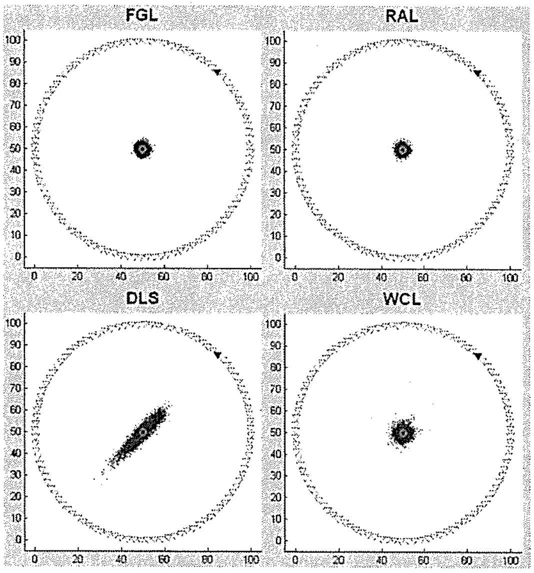

Figure 3. Point error circular beacon deployment

One sensor node represents the centroid. The remaining assumptions were as in the previous case. All simulations have been repeated 10000 times to eliminate empirical influences. FGL works iteratively using Taylor series for linearisation and coarse position estimates as start values. In our simulations we used the exact position of the unknown as start value.

B. *Results*

The results of the first simulation are illustrated in Fig. 2. Here the localisation error is depicted over the number of repetitions. It can be seen that WCL does not create an error ellipse in the classical sense. Moreover, WCL results in overlapping systematic and random errors. Due to the fact that the distances will only indirectly be used as weights, WCL does not take the network geometry into account. All positions will be moved into the beacon perimeter. That means that all sensor nodes outside the perimeter result in extremely high position errors. For the first simulation, where the unknown is placed inside but close to the perimeter, WCL delivers position error of about 10.5%. (Fig. 4)

FGL, DLS and RAL work differently. They are solving over-determined systems of equations using the Least Squares Method but different techniques to linearise the non-linear equations. As FGL uses the true position it delivers the smallest localisation error of about 2.1%. RAL results in the same range of 2.7%. Even the error ellipses are comparable in size and direction of the axes, which means the largest point errors. DLS has the largest localisation error of about 5.9%. The error ellipse is strongly correlated with the lineariser as the large semi axes points in that direction, as can be clearly seen in Fig. 3.

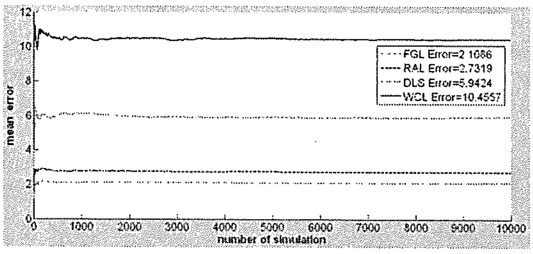

Figure 4. Mean error random beacon deployment

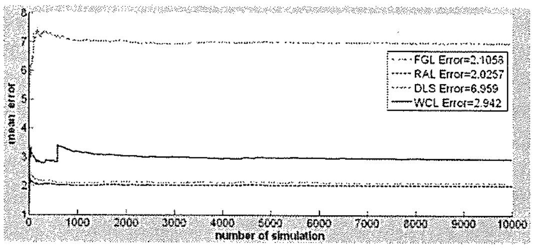

Figure 5. Mean error circular deployment

The second simulation (Fig. 3 and Fig. 5) has been chosen to investigate the influence of regularly positioned beacons with the same distances to the unknown sensor node. WCL delivers a higher accuracy of about 2.9%. This is due to the fact that all distances receive the same weight.

The fine-grained methods FGL and RAL reach the same accuracy of 2.1% and 2.0% respectively. DLS delivers 7%

where the large semi axis of the error ellipse points along the lineariser. In this special scenario, which is unlikely to occur in practice, DLS results in higher errors than WCL. Table 1 gives an overview over the achieved accuracy for all algorithms in both simulations.

TABLE I. ACCURACY REACHED BY DIFFERENT LOCALISATION METHODS

Method	Localisation Error			
	Random Beacon deployment Variance 10%	Circular Beacon deployment Variance 10%	Random Beacon deployment Variance 0%	Circular Beacon deployment Variance 0%
WCL	10.4%	2.9%	sys.-error	0.0
FGL	2.1%	2.1%	0.0	0.0
RAL	2.7%	2.0%	0.0	0.0
DLS	5.9%	6.9%	0.0	0.0

C. Discussion

Due to the minimal energy consumptions of coarse-grained algorithms, the "Weighted Centroid Localization" algorithm (WCL) is very promising for use in sensor networks. Because distance measurements are only used indirectly as input values, WCL delivers coarse positions. For the case that the unknowns are close to the centroid of the sensor field it may deliver exact position as it is based on centroid localisation. Due to the nature of randomly deployed wireless sensor networks this scenario is highly improbable. Therefore we recommend using WCL only, if a large number of beacons exists, if inaccurate position estimations with position errors larger than 10% are sufficient, or as starting values for a following refinement using distance measurements as observations.

"Fine-grained Localization" (FGL) is an iterative technique based on Taylor linearisation and uses position estimates as start values. As this method converges very quickly, only one iteration is sufficient for most cases. RAL and DLS convert the non-linear problem. This results in localisation methods where no iterations are required as no initial position as start values are used. For wireless sensor networks this is a huge advantage as no initial position for the unknowns is available in most cases.

As all methods are using the linear least squares model one may expect that all methods deliver the same accuracy. In our simulations, however, they differ if inaccurate measurements are used. In the theoretical case that the distance variance is zero, all algorithms result in the same accuracy as true values for the distances are used. Based on erroneous distance measurements the algorithms differ in their accuracy. FGL and RAL result in a similar localisation error, whereas DLS delivers greater position errors. This may be due to distance r_j which has an effect on all equations. This can be seen in Fig. 3. Choosing different beacons as lineariser, the resulting error ellipse follows with the large semi axes direction to the lineariser.

V. CONCLUSION

In this paper we investigated different methods for localisation in resource limited wireless sensor networks. Furthermore, three fine-grained techniques have been analysed. Here, the energetic effort is higher than for coarse-grained methods. However, as they solve over-determined systems of equations, this results in a higher positional accuracy. Even with a relatively high variance for the observed distances, all algorithms deliver position with a high accuracy. We showed that for special scenarios, e.g. where unknown sensor nodes are close to the centroid of the sensor field, WCL delivers fine-grained position estimates. For sensor nodes situated outside the beacon perimeter the resulting positions are useless as WCL does not take the network geometry into account and therefore pulls the sensor nodes into the beacon perimeter. Furthermore we showed that all fine-grained methods deliver good results. But due to the simple computation, only DLS and RAL are feasible for wireless sensor networks. Here, the equations in DLS are correlated with one beacon, which makes it prone to beacon failures. Moreover, for erroneous distance observations it results in high position errors. The equations in RAL are completely independent and therefore more robust to failures. This also results in a higher positional accuracy than DLS and WCL. In our simulations, FGL delivers the highest accuracy. But due to the high energy consumption, FGL is unsuitable for use in wireless sensor networks and should only be used on powerful base stations for post-processing.

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REFERENCES

Akyildiz, I. F., Su, W., Sankarasubramaniam, Y., and Cayirci, E. (2002). A survey on sensor networks. *IEEE Communications Magazine*, pp. 102-114.

Blumenthal, J., Reichenbach, F., and Timmermann, D. (2005). Precise positioning with a low complexity algorithm in ad hoc wireless sensor networks. *PIK - Praxis der Informationsverarbeitung und Kommunikation 28* (pp. 80–85)

Born, A., Reichenbach, F., Bill, R., and Timmermann, D. (2006). Bestimmung Optimaler Startwerte zur Exakten Lokalisierung mittels Geodätischer Ausgleichung. *Technischer Bericht 2006/07, 5. GI/ITG KuVS Fachgespräch*. Stuttgart Juli 2006, Deutschland, 2006. , S. 93 - 98.

Born, A. and Bill, R. (in press) Reducing the Calculation for Precise Localization in Wireless Sensor Networks. *Proceedings of the IEEE Vehicular Technology Conference* (VTC 2010–Spring)

Bulusu, N. (2000). Gps-less low cost outdoor localization for very small devices. *IEEE Personal Communications Magazine*. 7, 28–34.

He, T., Huang, C., Blum, B. M., Stankovic, J. A., and Abdelzaher, T. (2003). Range-free localization schemes for large scale sensor networks. *Proceedings of the 9th annual international conference on Mobile computing and networking*, San Diego, CA, USA, (pp. 81-95), 2003

Savvides, A., Han, C. C., and Strivastava, M.B. (2001). Dynamic fine grained localization in ad-hoc networks of sensors. *Proceedings of the 5th International Conference on Mobile Computing and Networking*, Mobicom 2001, (pp. 166-179), Rome, Italy, July 2001.

Reichenbach, F., Born, A., Timmermann, D. and Bill, R. (2006. Splitting the linear least squares problem for precise localization in geosensor networks. *Proceedings of the 4th International Conference on GIScience*.