

# **Vertical Error Propagation in Digital Surface Model Differencing**

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## **Abstract**

*One of the most fundamental concepts in digital terrain analysis is calculating change through time. This paper presents a new methodology for accounting for vertical error in digital surface model (DSM) differencing between two co-located rasters for different points in time. The vertical errors are stored in an "error raster", in which each pixel value represents the vertical error at that location for the associated DSM. A vertical buffer is generated around two surfaces. The vertical error for each pixel is determined by the pixel value in the error raster, which is either added to or subtracted from the original surface. The two buffered surfaces are then differenced using a series of logical operators. The vertical buffer method in this paper is a more appropriate method of examining spatial change through time because it propagates error to the resulting change surface, and tests whether the observed change is conflated with errors from either of the DSMs.*

**Keywords:** error propagation, DEM, change analysis, geostatistics, coastal geomorphology

## **1. Introduction**

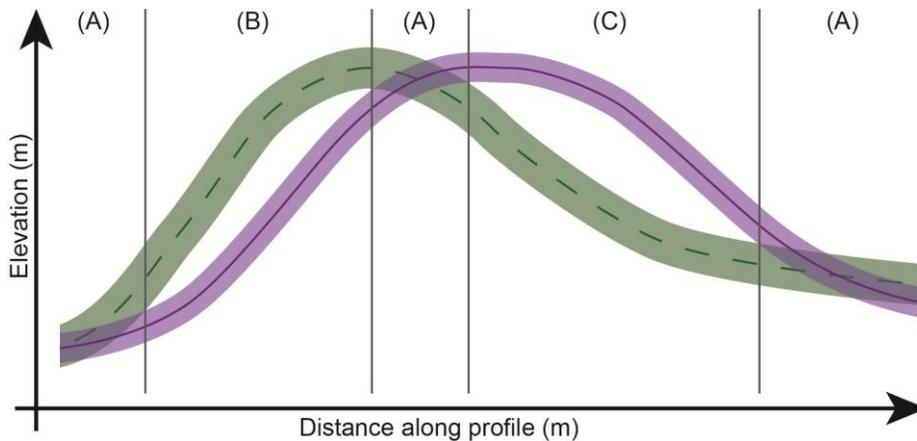
One of the most fundamental concepts in digital terrain analysis is calculating change through time. Change analysis is typically done in a geographic information system (GIS) and the results of this analysis provide insight into spatiotemporal patterns that are integral to many fields of study. In coastal geomorphology, engineering, and management the surface change analysis yields important insight into patterns and processes such as erosion and deposition (Theuerkauf and Rodriguez 2012, Eamer and Walker 2013). This information is useful to coastal managers because it can be used to assess barrier island response and recovery to storms and the likelihood of coastal inundation due to sea-level rise (Gesch 2009). Surface change analysis also provides valuable information that can help predict the success of coastal nourishment projects and response and recovery to storms (Houser, Hobbs and Saari 2008). Although results of DSM differencing are useful, the resulting change surface can be greatly affected by spatial error.

Vertical error quantification and propagation in digital surface models (DSMs) has received relatively little attention, despite the important implications of not accounting for error. In the case of digital elevation models (DEMs), a small amount of error in the vertical dimension can cause the resulting change surface to appear significant, when the observed change is, in fact, conflated with error. While the horizontal error can be more easily modelled, the vertical error is much more difficult to quantitatively address by coordinate transformations and modelling.

The purpose of this paper is to present a method for quantitatively accounting for vertical errors in DSM differencing. The proposed methodology uses a vertical buffer to represent the error in DSMs. The two surfaces are then subtracted using Boolean Logic operations to calculate the surface change while accounting for the vertical error. This relatively simple method is adaptable to a wide variety of surface models and applications, and provides information about where the change is conflated with error. The feasibility of this method is demonstrated using Follets Island, TX as a coastal geomorphology case study.

## 2. Methodology

The proposed methodology employs vertical buffers to quantitatively represent the vertical error in a DSM. Each DSM has an accompanying “error raster” with co-registered pixels. In this raster, the value of each pixel represents the magnitude of error at that location. The spatial structure of the values in the error raster can be isotropic or anisotropic, depending on the nature of the error and factors affecting it. The error raster for each DSM is used to generate a vertical buffer around the original DSM (Figure 1). The advantage of using an entire raster to represent this error instead of a single value for an entire DSM is that the raster approach enables the error to vary spatially. This vertical buffering is performed on two DSMs, one representing the surface at time 1 and the other representing the same area at time 2. The buffered area represents the 95% probability that the true value will fall within. When the two buffered DSMs for times 1 and 2 are differenced using Boolean logic (Equations 1, 2, and 3), the resulting change surface represents the 95% probability that the two surfaces are not overlapping purely due to error.



**Figure 1:** Conceptual model of DSM differencing with vertical error propagation. The solid line and dashed line represent the same surface at two points in time, with the dashed green line as time 1 and the solid purple line as time 2. The transparent area around each line represents the vertical uncertainty of the DSM. For simplicity, this conceptual model uses an isotropic error field to each of the surfaces. The letters along the top correspond to different change calculation scenarios: (A) corresponds to areas where the observed difference is conflated with error, (B) corresponds to areas of erosion from time 1 to time 2, and (C) corresponds to areas of accretion/deposition from time 1 to time 2.

$$(A_{X \pm ERROR} \cap B_{X \pm ERROR}) \rightarrow C = Null \quad (1)$$

$$(A_{X-ERROR} > B_{X+ERROR}) \rightarrow C = B_{X+ERROR} - A_{X-ERROR} \quad (2)$$

$$(A_{X+ERROR} < B_{X-ERROR}) \rightarrow C = B_{X-ERROR} - A_{X+ERROR} \quad (3)$$

In Equations 1, 2, and 3,  $A_{X-ERROR}$  and  $B_{X-ERROR}$  represent the surfaces of rasters A and B, respectively, with the error subtracted from the surface. Similarly,  $A_{X+ERROR}$  and  $B_{X+ERROR}$  represent the surfaces with the error added. The calculated change surface value for the pixel is given by C, and Null is used to denote cases when the change surface value is set to the null value. Applying these operators to surfaces A and B, the resulting change raster will contain positive, negative, and null values. The change raster can be interpreted as the 95% probability that the change in elevation is not conflated with the errors that have been accounted for.

Equation 1 represents areas where the two vertical buffers overlap and it is likely that the observed change is conflated with error. Equation 2 is characterized by a negative value and indicates areas of potential erosion. These are areas where the surface at time A has lowered to the value at time B (red hues in Figure 2). Positive values (Equation 3; blue hues in Figure 2) are indicative of areas where the surface at time B is higher than time A. Equation 3 suggests depositional processes have occurred. Figure 1 illustrates each of these cases, with the green line representing the surface at time A and the purple representing the surface at time B. The letters in parentheses along the top of the figure represent the case that applies in that location. Although this figure presents a simplistic version of the vertical buffering where the buffer width is uniform for a given surface, variation in the error surface and error bands does not affect the fundamental concept of the outlined DSM differencing.

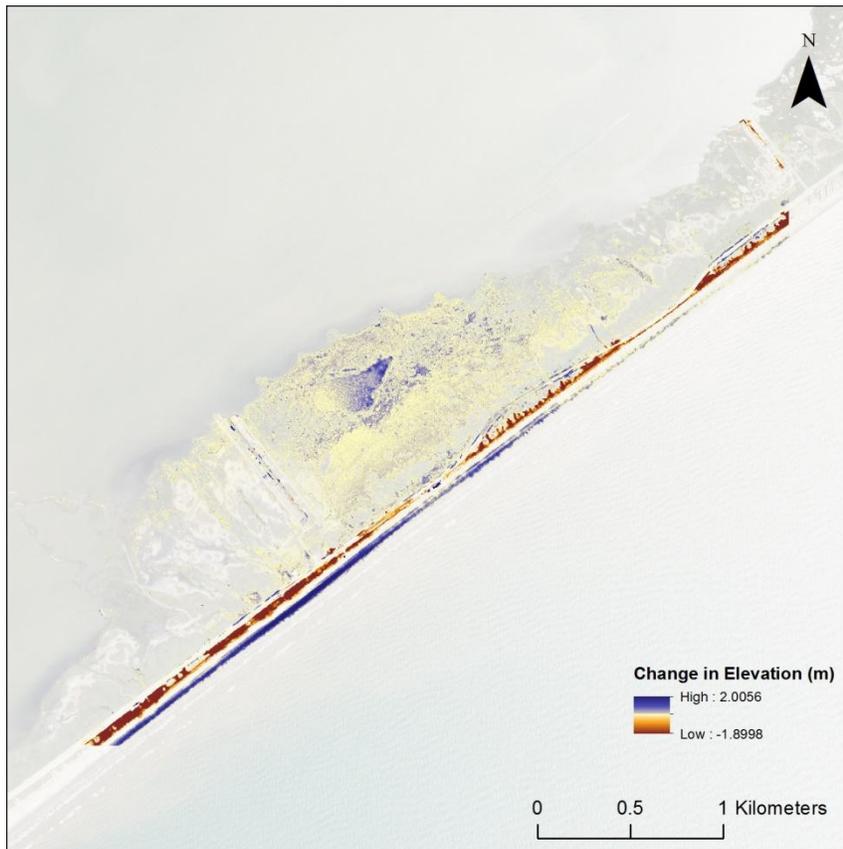
The DSM differencing algorithm was written in python and employs the geospatial data abstraction library (GDAL) module. The GDAL module enables the script to read the geographic information in the header and access the pixel values, which can then be added or subtracted to produce the output change surface. The change surface is then written out to a new raster image (Figure 2) with an updated geographical header. This python program is also available as a standalone executable file. The advantage of the single executable file is that it contains the built-in necessary functions from the GDAL module and it is more interoperable to various operating systems.

### **3. Results**

The vertical buffer approach to DSM differencing (Figure 2) does not overestimate the change from time 1 to time 2. Traditional DSM differencing returns an elevation change for every pixel (Figure 3), regardless of whether the observed change in elevation is less than the error of the two individual DSMs. Applying traditional simple subtraction, the total area with non-null change values was over 4.2 square kilometers. When the vertical buffer approach was applied, the total area with significant change was reduced to just over 1.5 sq. km. This reduction in area is attributed to areas where the calculated change is less than the error in one or both of the input DSMs.

In addition to reducing the total area with significant change, applying vertical buffering to DSM differencing also reduced the mean, range, and standard deviation of the observed change. The mean change was approximately 0.17 m lower using the vertical buffer approach, compared to the traditional differencing. The range was

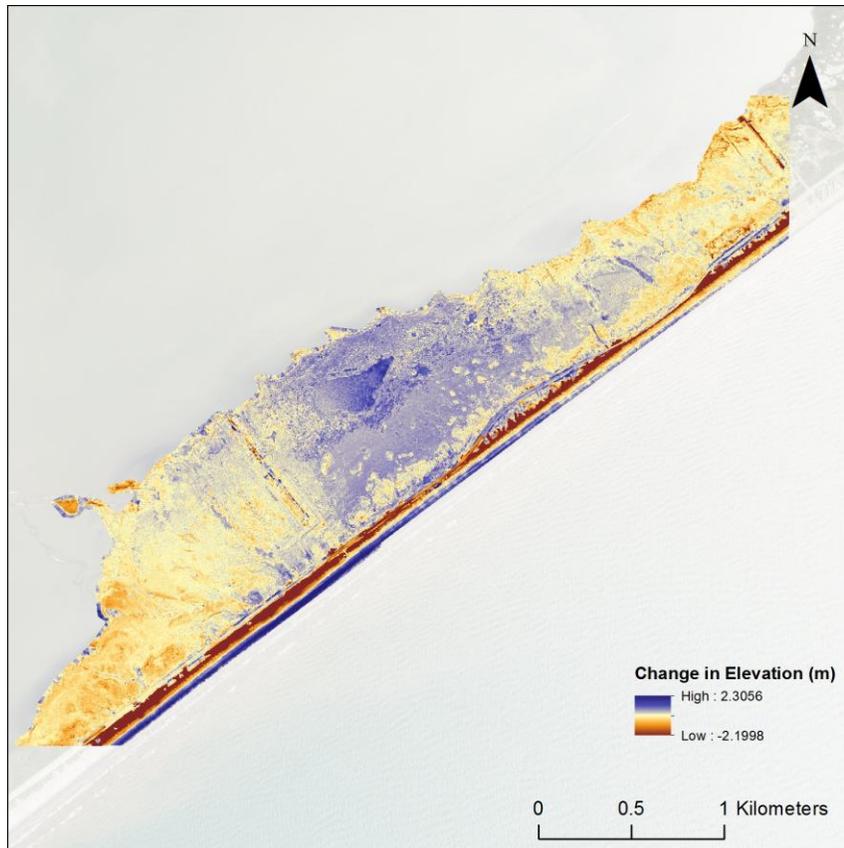
reduced from 4.5 m to 3.9 m, and the standard deviation was reduced from 0.27 m to 0.24 m.



**Figure 2:** Vertical buffer approach to DSM differencing results in a more conservative assessment of changes in the DSM. Negative values (red hues) represent areas where the surface has decreased from time 1 to time 2, and positive values (blue hues) represent areas where the surface has increased in elevation from time 1 to time 2.

#### **4. Conclusions**

The Follets Island case study demonstrates that vertical error propagation greatly affects the resulting change surface. Figure 2 represents the vertical buffer approach to DSM differencing where vertical errors are propagated to the resulting change surface. When this error is accounted for, the overall magnitude and spatial distribution of change is reduced, compared to the traditional method of DSM differencing (Figure 3), where the surface at time 1 is simply subtracted from the surface at time 2. Changes included in Figure 3 but not in Figure 2 represent the areas where the observed change from time 1 to time 2 was conflated by the vertical error in the two input DSMs.



**Figure 3:** The traditional DSM differencing approach overestimates the magnitude and coverage of significant change across the DSM. Negative values (red hues) represent areas where the surface has decreased from time 1 to time 2, and positive values (blue hues) represent areas where the surface has increased in elevation from time 1 to time 2.

It is feasible to use the proposed vertical buffer approach to account for vertical errors in surface change analysis. Since surface change analysis is used extensively as a decision support tool, it is important to account for these vertical errors. The proposed pixel-based approach is extremely efficient at processing large raster datasets and is highly customizable. The high portability and efficiency of this processing makes it highly useful for researchers, planners, developers, and managers as a way of accounting for error in their analysis and ultimately improve the accuracy of the final products.

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