3. Actual error propagation analysis

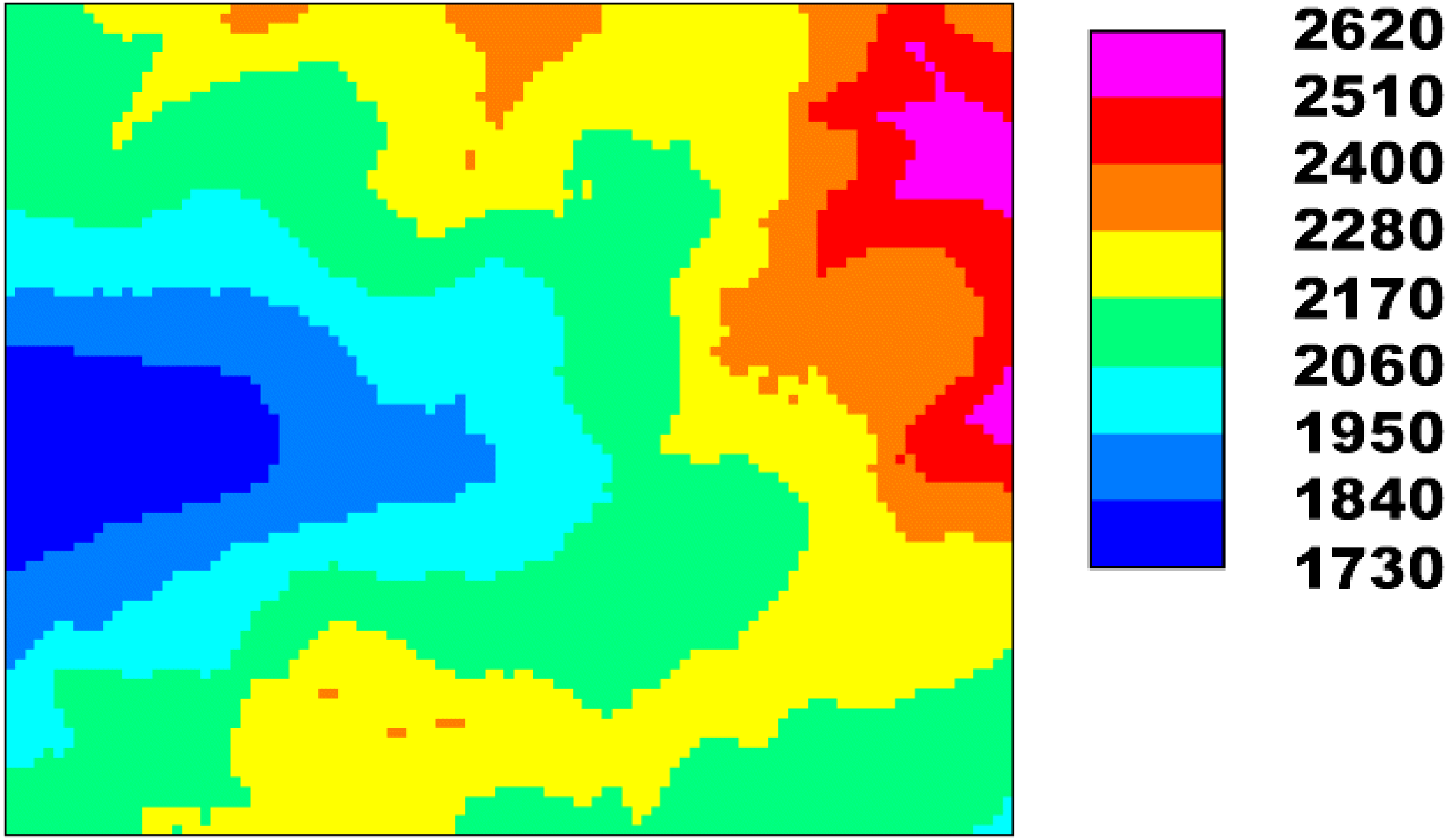
We will discuss two methods:

- Taylor series approximation (yesterday)
- Monte Carlo method (today)

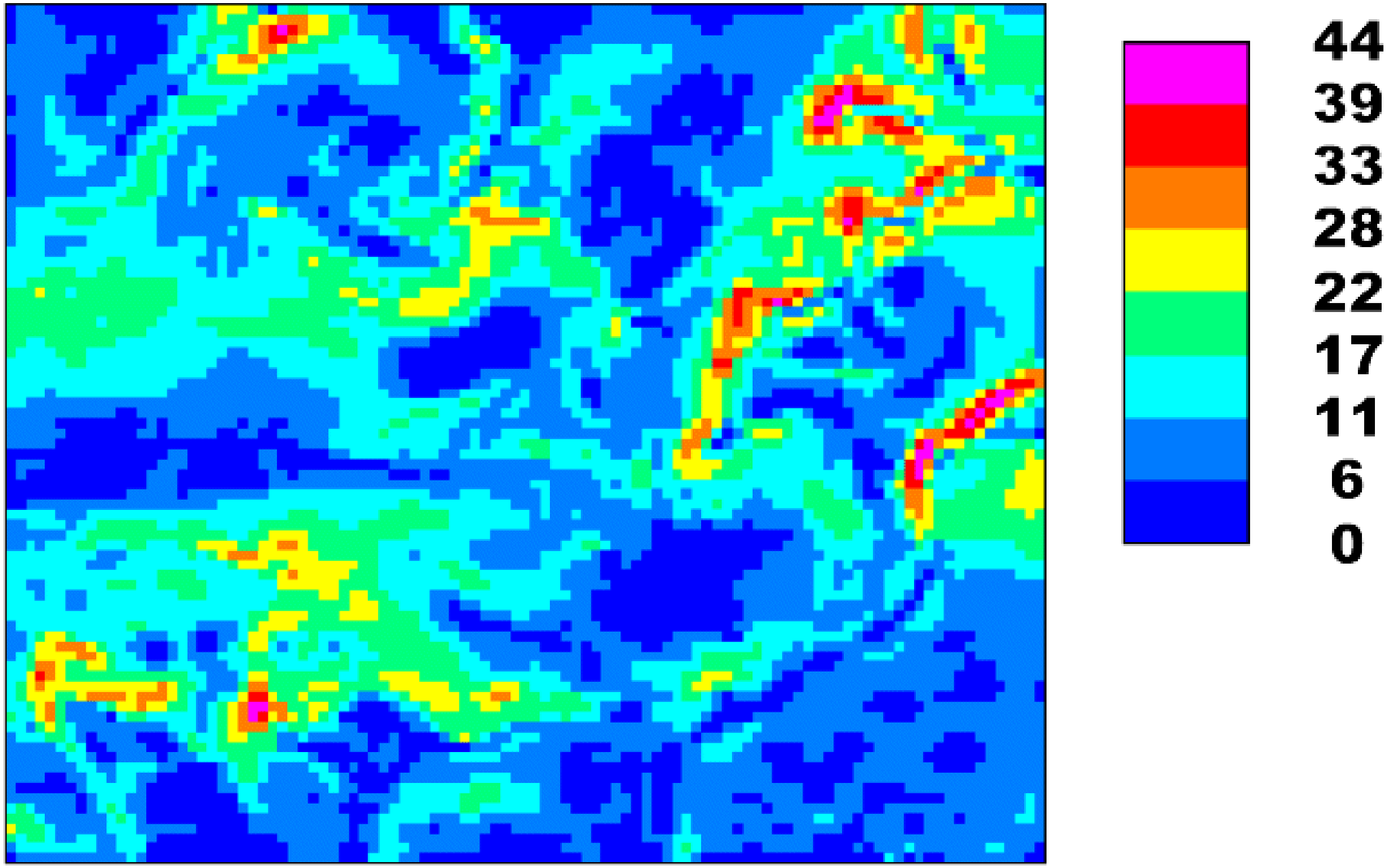
Monte Carlo method

Introduce by means of an example

Example: computing slope from DEM for a 2 by 2.5 km area in the Austrian Alps

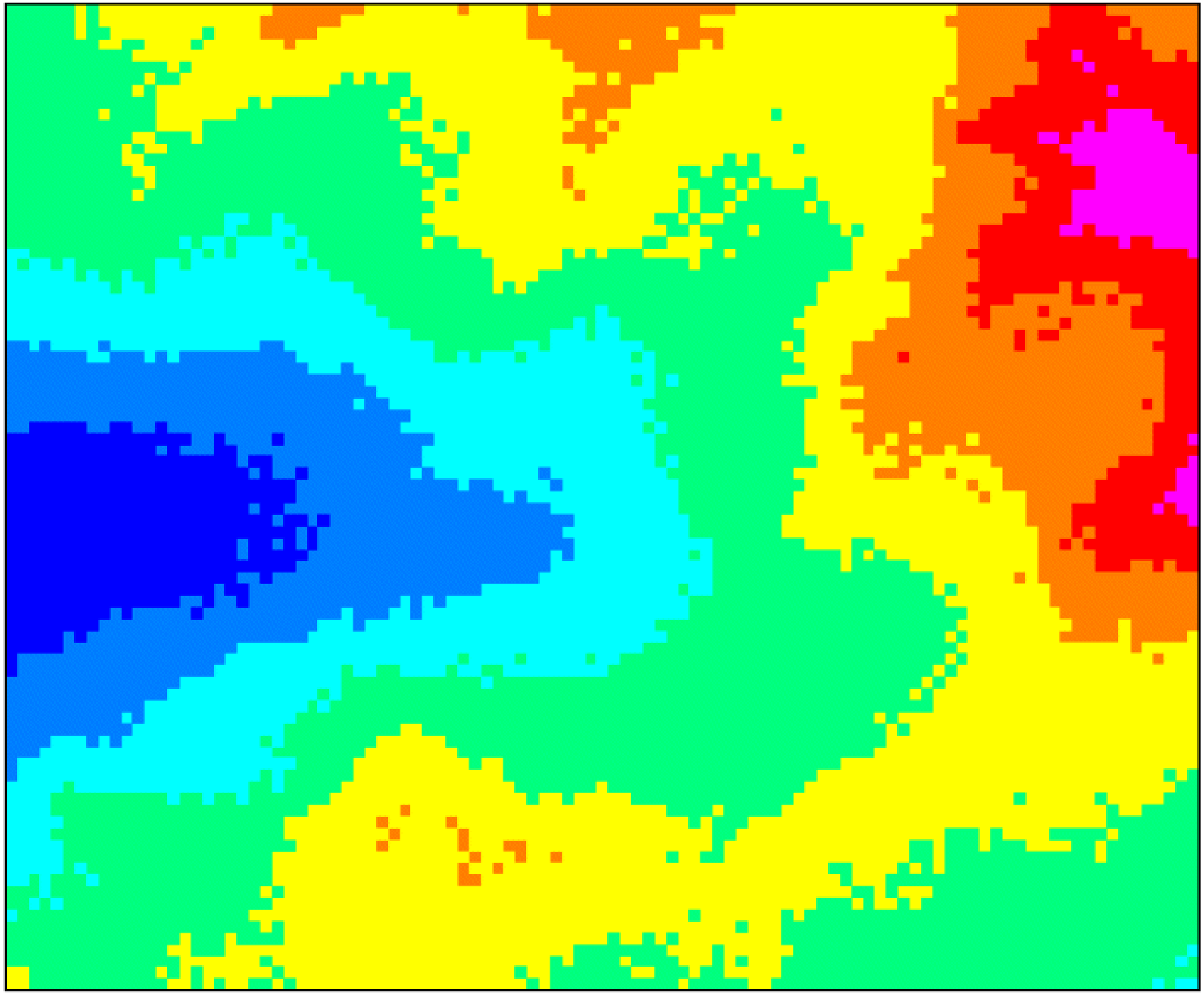


Slope map computed from the DEM (percent):

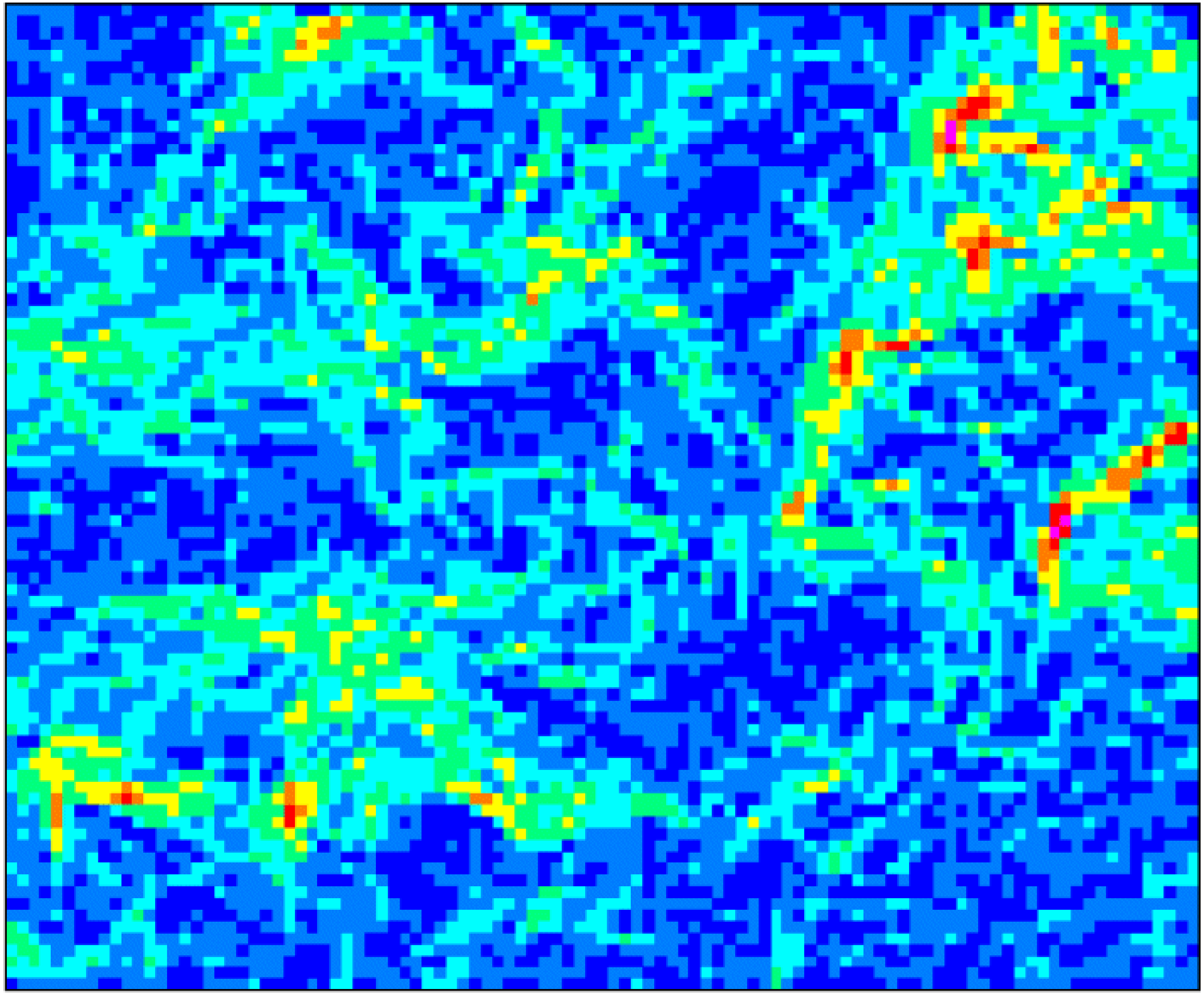


Now let the error in the DEM be $\pm$10 meter

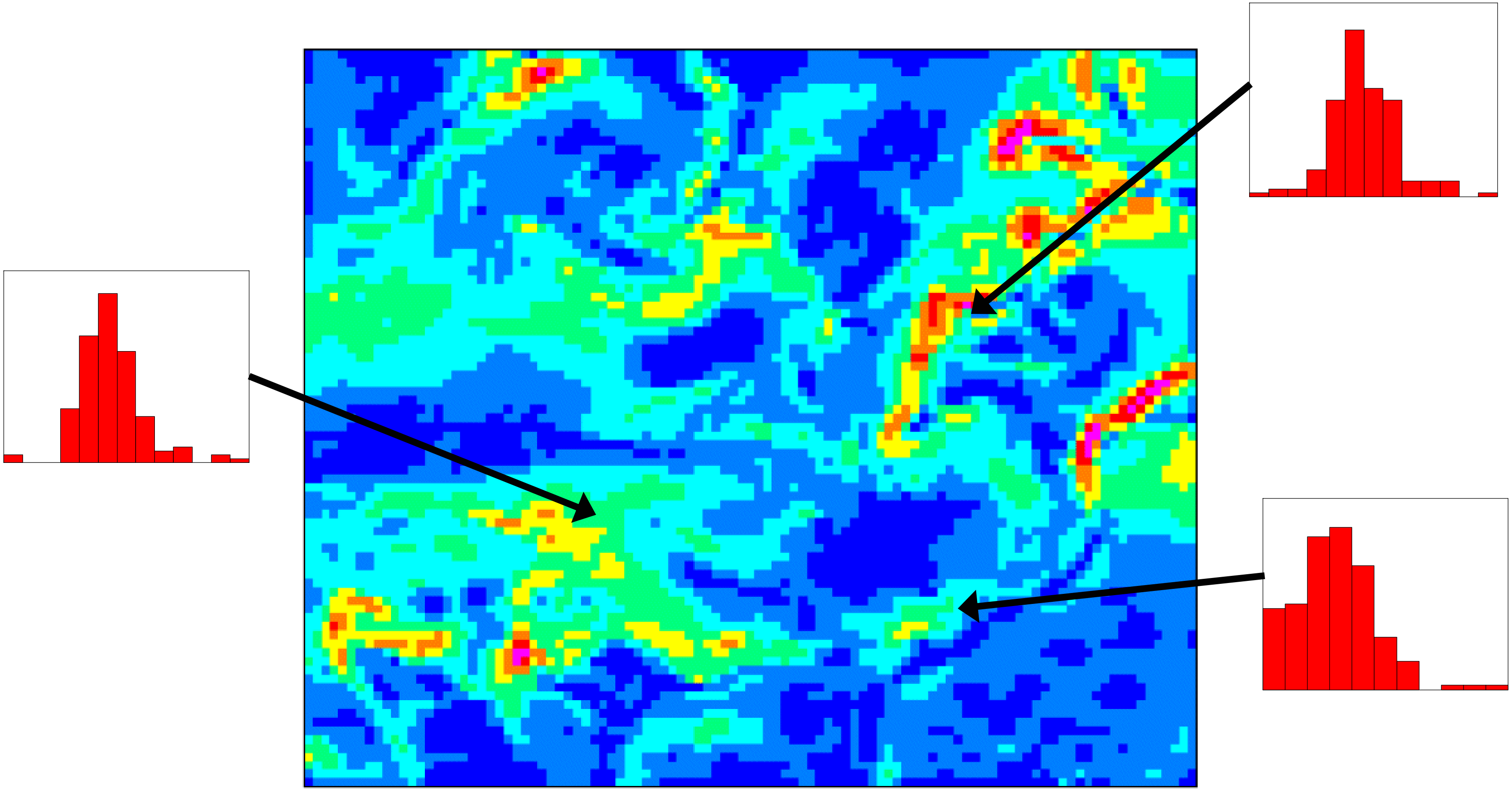
Realisations of uncertain DEM:



Corresponding uncertain slope maps:



Histograms capture uncertainty in slope:



Summary of Monte Carlo method

- Repeat many times (>100):
  - Simulate a possible reality from the probability distribution of the uncertain inputs
  - Run model with simulated input and store result
- Compute and report statistics of the sample of outputs (e.g. mean, standard deviation, proportion that exceeds critical threshold)

EXERCISE 4

Stochastic spatial simulation

Kriging makes optimal estimates: it yields the most likely value at any location

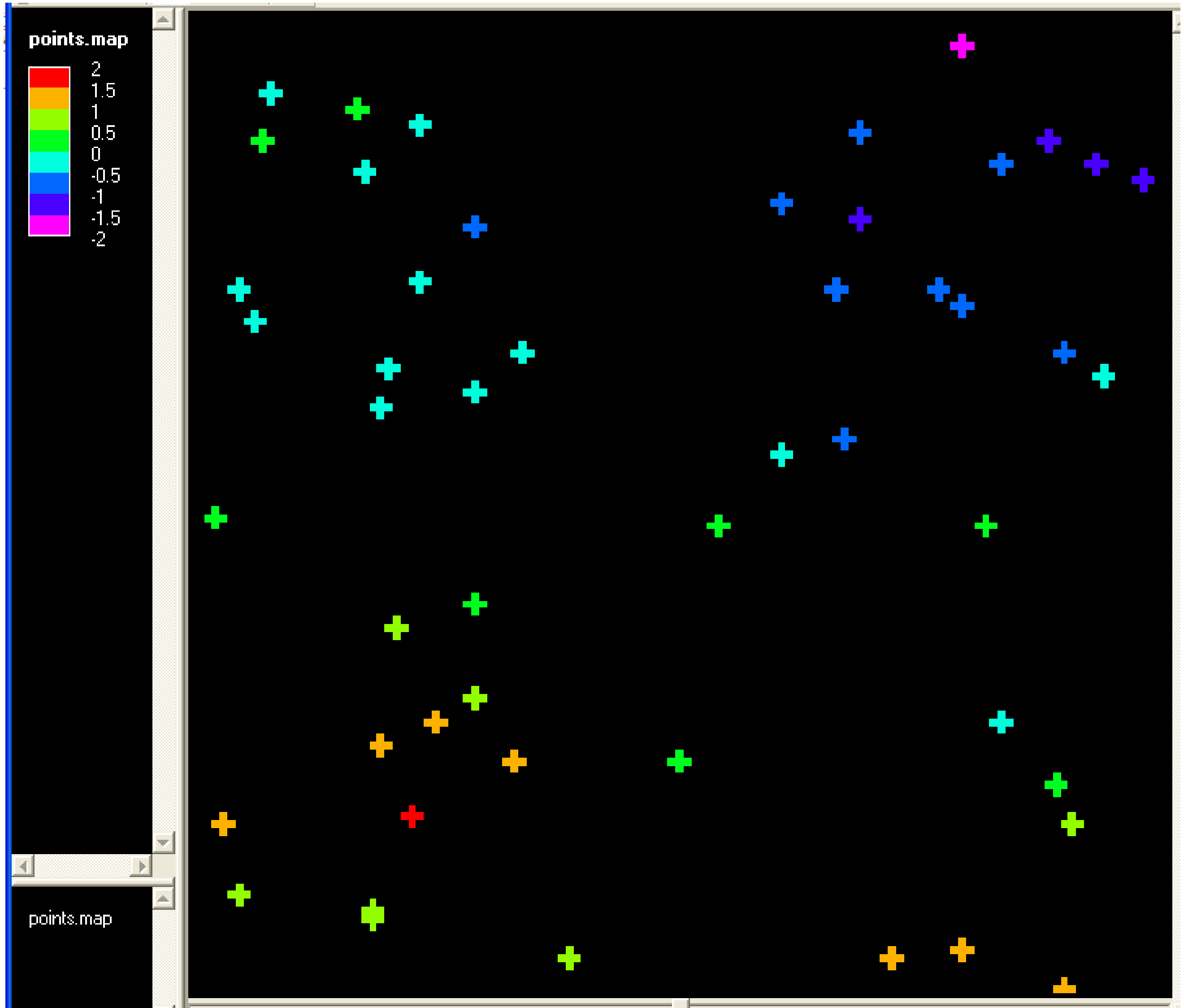
But it is only an estimate. The real value is uncertain, we treat it as stochastic, it fluctuates around the kriged value in an unpredictable way

In stochastic spatial simulation we do not compute an estimate but instead we generate a possible reality, by simulating from the probability distribution (using a random number generator)

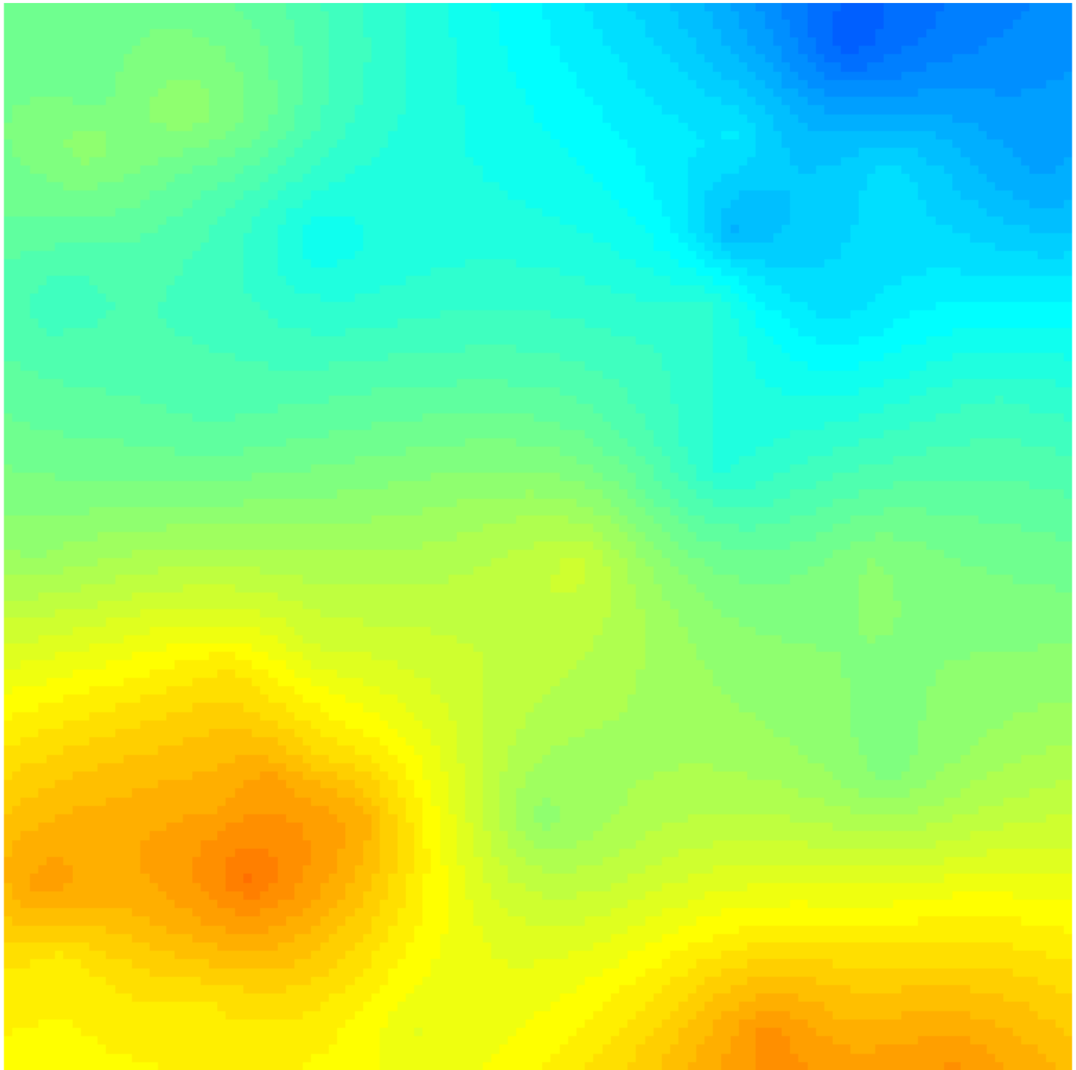
Sequential Gaussian Simulation

1. Visit a randomly chosen grid cell
2. Compute kriging estimate and kriging standard deviation for this cell
3. Sample from the kriging probability distribution using a pseudo-random number generator and assign the simulated value to the grid cell
4. Add the simulated value to the data set
5. Repeat procedure by visiting a new randomly chosen grid cell, until all grid cells have been visited

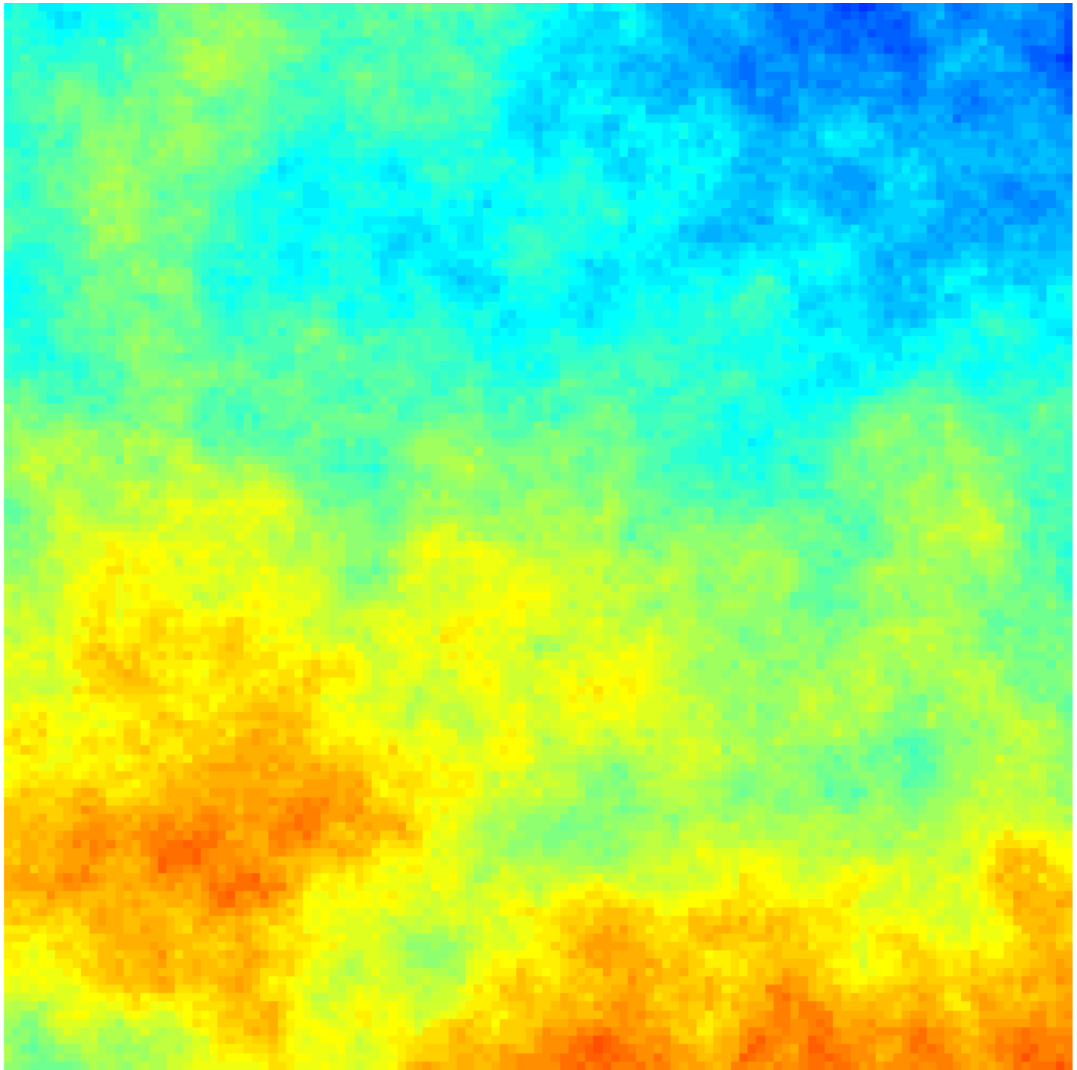
Illustrate differences between kriging and simulation with an example



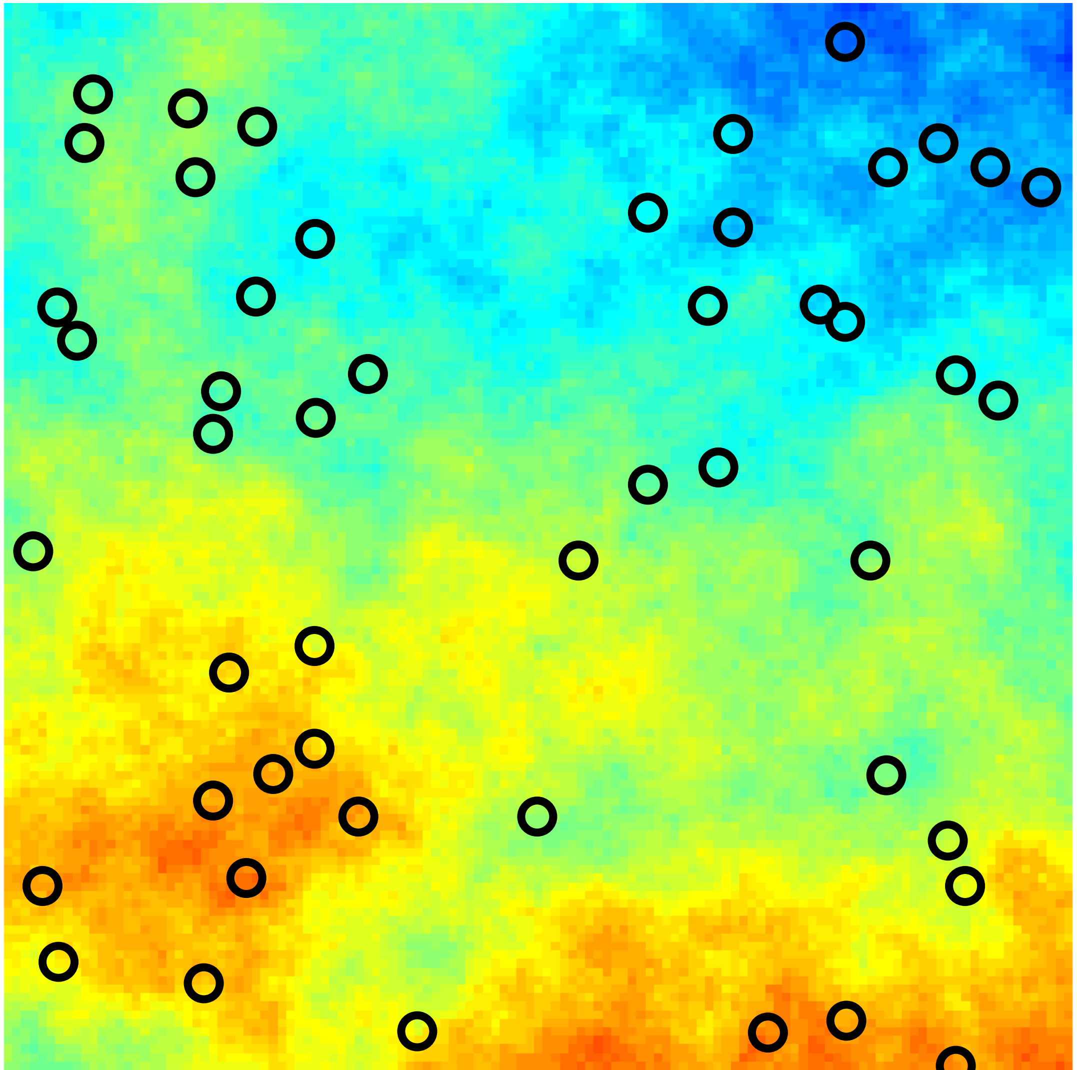
Kriged map, interpolated from the 50 observations



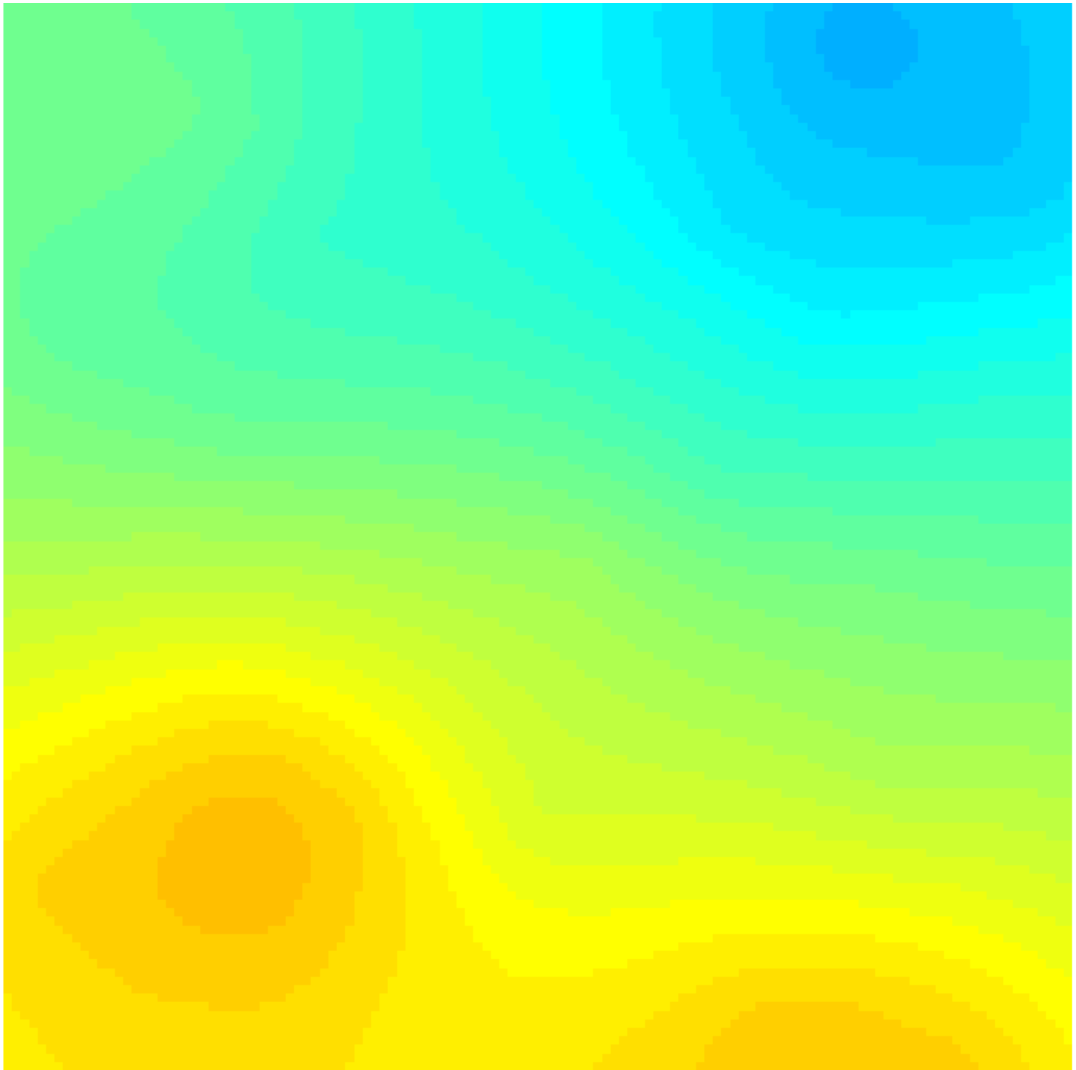
Simulations, obtained by sampling from the probability distribution using sequential Gaussian simulation



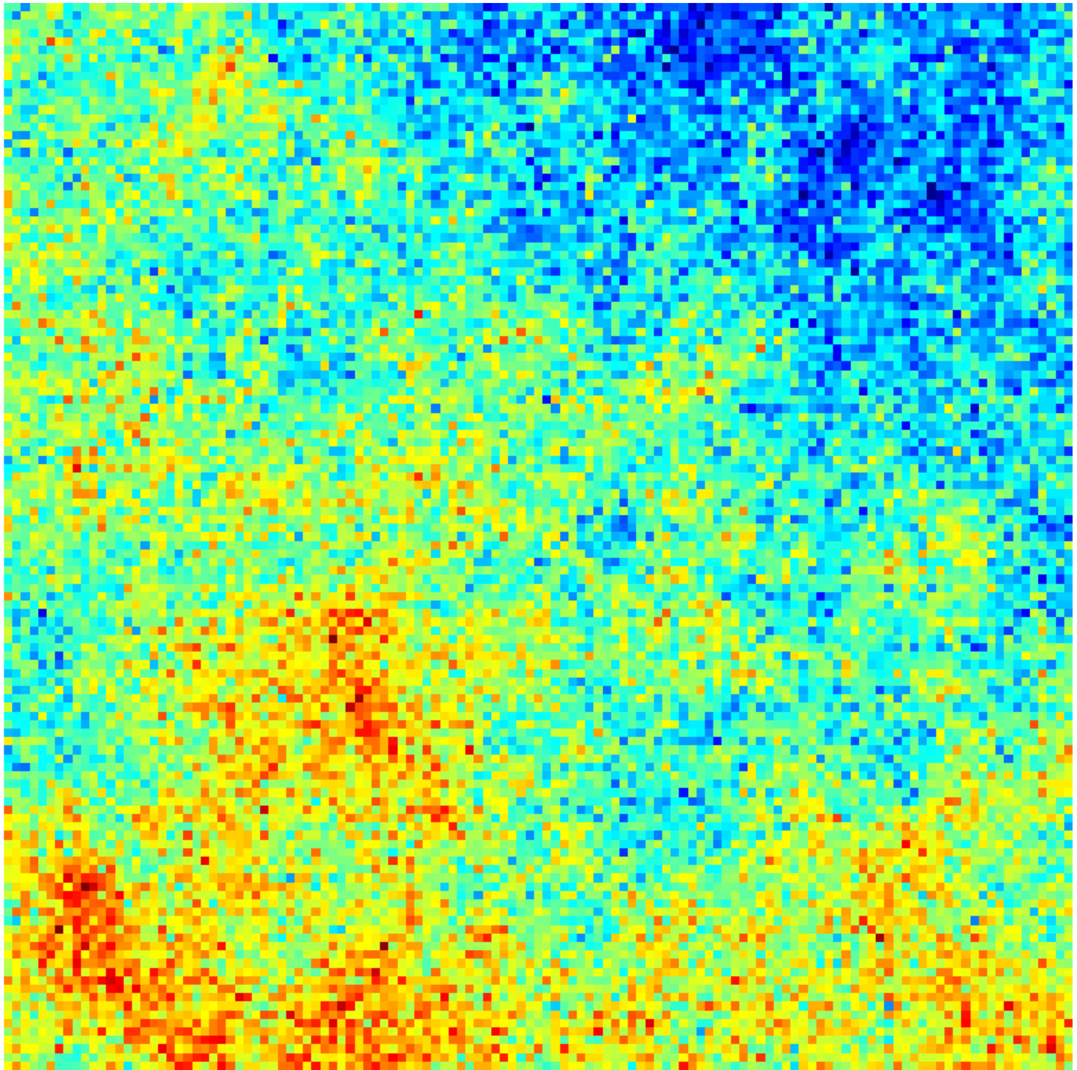
Simulations, with data points: we used conditional simulation!



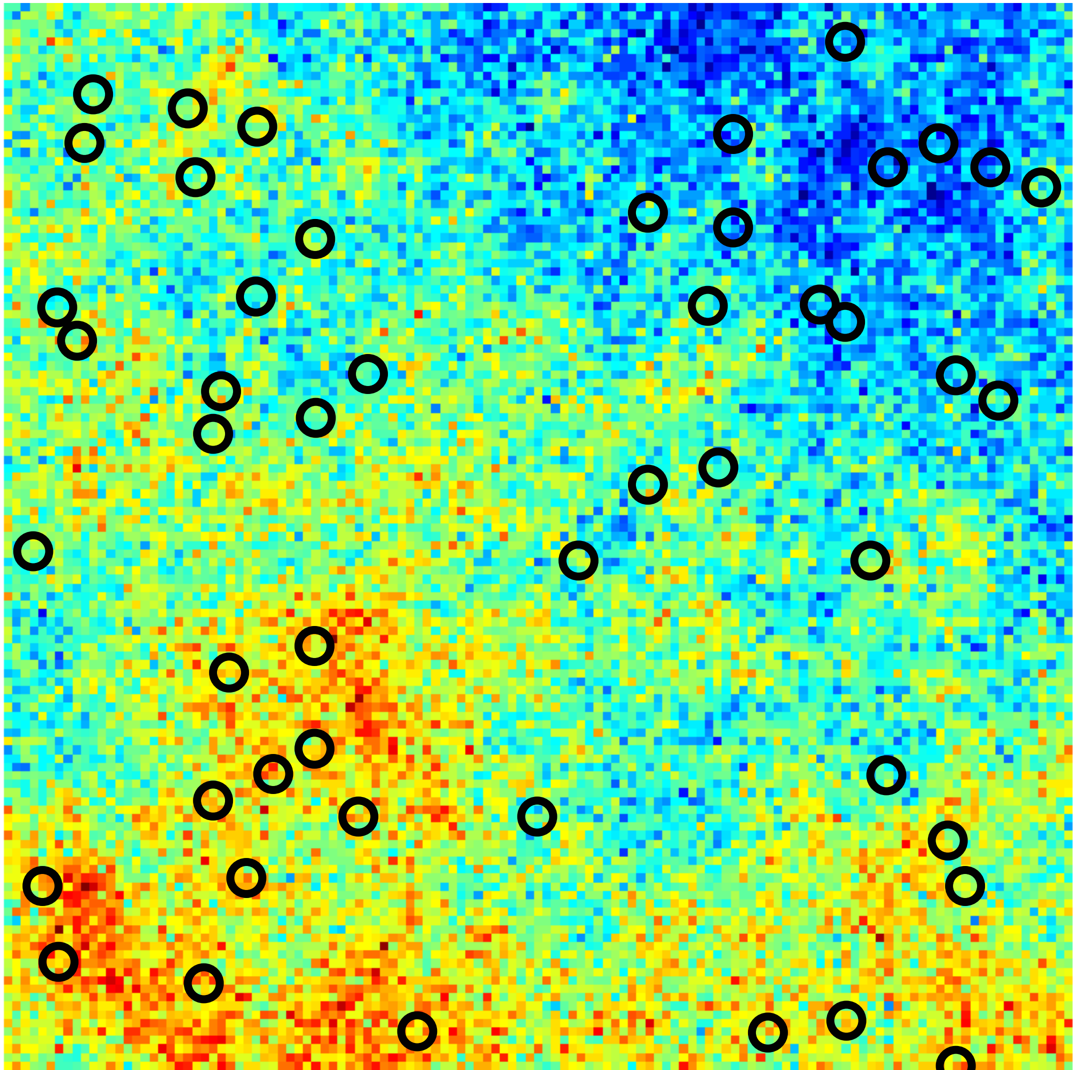
Kriged map, case of weak spatial correlation



Simulations, case of weak spatial correlation



Simulations, case of weak spatial correlation

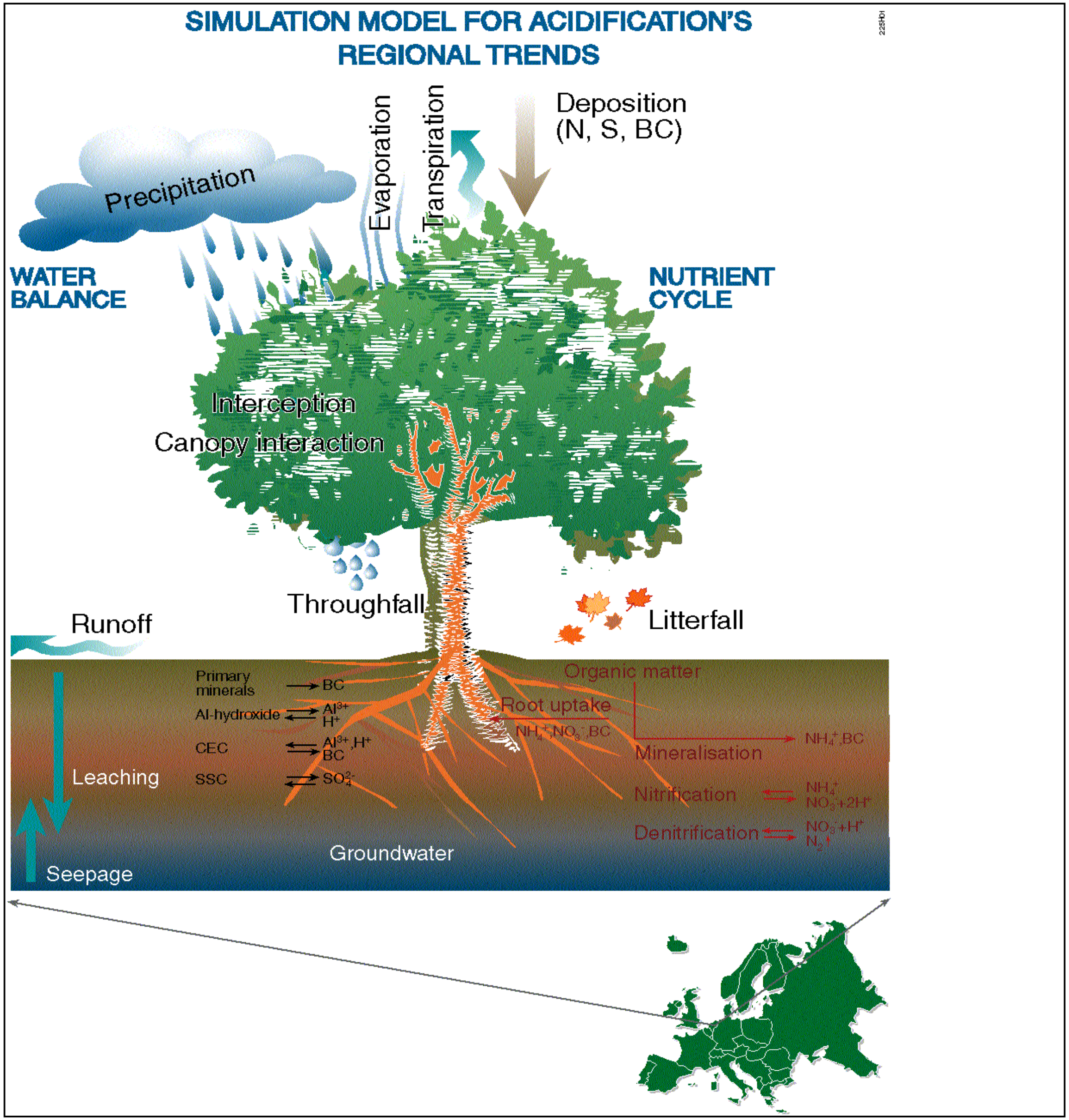


Few slides on a real-world uncertainty propagation example: uncertainty propagation in a soil acidification model (see also journal article in course notes)

SMART2

soil acidification model

Kros et al., J. Env. Quality 28, 366-377



Only two uncertain *categorical* inputs to SMART2:

- soil type: (only) 7 types
- vegetation: (only) 4 types
- soil&vegetation: $7 \times 4 = 28$ types
- 28 indicator variograms
- 378 cross-variograms

There is no way that these variograms can all be fitted properly and used in the uncertainty analysis, some simplifications had to be made

Nineteen uncertain *continuous* inputs to SMART2:

- 11 soil-related variables: C/N ratio organic matter, weathering rates, nitrification and denitrification fractions, etc.
- 8 vegetation-related variables: forest filtering parameters, dry deposition factor, transpiration, N content in shoot/stem, etc.
- 19 times distribution type and variogram required (cross-correlation errors ignored)
- and do so for each soil/vegetation type separately!

Uncertainty sources

Table 6 Distributions of model parameters

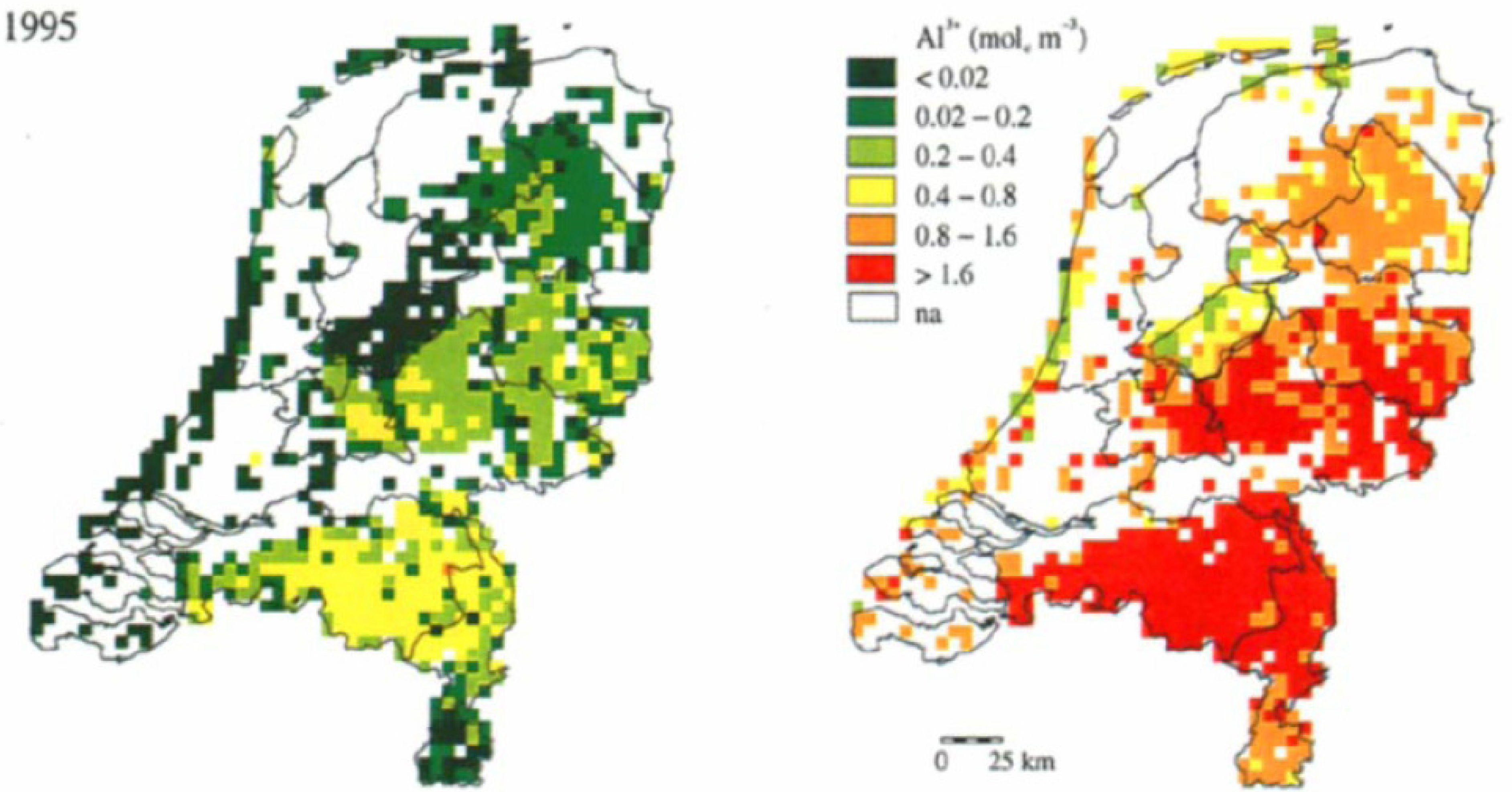
Parameter	Unit	Distribution type	Mean	SD	Min.	Max.
$fNH_{3\ fis}$	-	uniform	0.1	-	0.0	0.2
$f_{pr}NH_{4\ rn}$	-	uniform	1.5	-	0.0	2.0
kCa_{fe}		normal	0.14	0.05	0.08	0.23
kMg_{fe}	a^{-1}	normal	0.26	0.04	0.19	0.31
kK_{fe}	a^{-1}	normal	0.22	0.08	0.09	0.32
k_{rd}	a^{-1}	uniform	2.0	-	1.5	2.5
$k_{ni\ 0}$	a^{-1}	uniform	85.0	-	50.0	120.0
$k_{ni\ 1}$	a^{-1}		20.0	-	10.0	30.0
$k_{ni\ 2}$	a^{-1}	uniform	2.0	-	0.0	5.0
$k_{ni\ 3}$	a^{-1}	uniform	2.0	-	0.0	5.0
k_{pr}	a^{-1}	uniform	50.0	-	25.0	75.0
$kK/Na_{ue\ pm}$	a^{-1}	uniform	$1.1 \cdot 10^{-4}$	-	$2.0 \cdot 10^{-5}$	$2.0 \cdot 10^{-4}$
$kCa/Mg_{ue\ pm}$	a^{-1}	uniform	$5.25 \cdot 10^{-4}$	-	$5.0 \cdot 10^{-5}$	$1.0 \cdot 10^{-3}$
$kAl_{ue\ ac\ 1}$	a^{-1}	uniform	0.02	-	0.01	0.03
$kAl_{ue\ ac\ 2}$	a^{-1}	uniform	0.20	-	0.10	0.30
$kAl_{ue\ ac\ 3}$	a^{-1}	uniform	0.20	-	0.10	0.30
KAl_{ox}	$(mol\ l^{-1})^{-2}$	uniform	$10^{8.77}$	-	$10^{8.11}$	$10^{9.35}$
$KH_{ex\ 0}$	$(mol\ l^{-1})^{-1}$	lognormal	3.50 (33.0)[1]	1.00	1.50 (4.5)	5.50 (2.5)
$KH_{ex\ 1}$	$(mol\ l^{-1})^{-1}$	lognormal	6.79 (889.0)	1.29	4.76 (117.0)	9.19 (9800.0)
$KH_{ex\ 2}$	$(mol\ l^{-1})^{-1}$	lognormal	7.83 (2515.0)	1.96	4.01 (55.0)	10.3 (2533.0)
$KH_{ex\ 3}$	$(mol\ l^{-1})^{-1}$	lognormal	7.72 (2252.0)	3.52	2.62 (14.0)	12.2 (98789.0)
$KAl_{ex\ 0}$	$mol\ l^{-1}$	lognormal	4.30 (74.0)	1.00	2.30 (10.0)	6.30 (545.0)
$KAl_{ex\ 1}$	$mol\ l^{-1}$	lognormal	-1.18 (0.3)	1.32	-2.98 (0.1)	0.9 (0.6)
$KAl_{ex\ 2}$	$mol\ l^{-1}$	lognormal	-0.18 (0.8)	1.38	-2.19 (0.1)	1.6 (0.1)
$KAl_{ex\ 3}$	$mol\ l^{-1}$	lognormal	-0.14 (0.9)	1.44	-1.80 (0.2)	2.7 (5.5)
$KNH_{4\ ex\ 0}$	$(mol\ l^{-1})^{-1}$	lognormal	1.40 (4.1)	0.20	1.00 (2.7)	1.80 (6.0)
$KNH_{4\ ex\ 1}$	$(mol\ l^{-1})^{-1}$	lognormal	1.69 (5.4)	1.33	-0.5 (0.6)	3.58 (36.0)
$KNH_{4\ ex\ 2}$	$(mol\ l^{-1})^{-1}$	lognormal	5.23 (187.0)	1.91	1.64 (5.2)	8.06 (3165.0)
$KNH_{4\ ex\ 3}$	$(mol\ l^{-1})^{-1}$	lognormal	7.62 (2039.0)	1.93	4.79 (120.0)	10.6 (3478.0)
$KK_{ex\ 0}$	$(mol\ l^{-1})^{-1}$	lognormal	2.50 (12.0)	0.70	1.10 (275.0)	3.90 (49.0)
$KK_{ex\ 1}$	$(mol\ l^{-1})^{-1}$	lognormal	3.60 (37.0)	0.69	2.40 (11.0)	4.4 (6.0)
$KK_{ex\ 2}$	$(mol\ l^{-1})^{-1}$	lognormal	4.77 (118.0)	0.63	3.61 (37.0)	5.5 (52.0)
$KK_{ex\ 3}$	$(mol\ l^{-1})^{-1}$	lognormal	6.17 (478.0)	0.87	5.02 (151.0)	7.4 (772.0)
$KSO_{4\ ad}$	$m^3\ mol_c^{-1}$	lognormal	0.00 (1.0)	1.15	-2.30 (0.1)	2.3 (10.0)

[1]) For lognormal distributions, values in brackets denote the nominal values; the other values concern the log-transformed counterparts.

You can imagine how much work the uncertainty analysis involved, and yet:

- only part of all uncertain inputs were considered
- no attention was paid to model error (errors in model structure, less important processes ignored, etc.)

Lower and upper limit of 90% prediction interval of aluminium concentration at 1 m depth



Uncertainty in pH

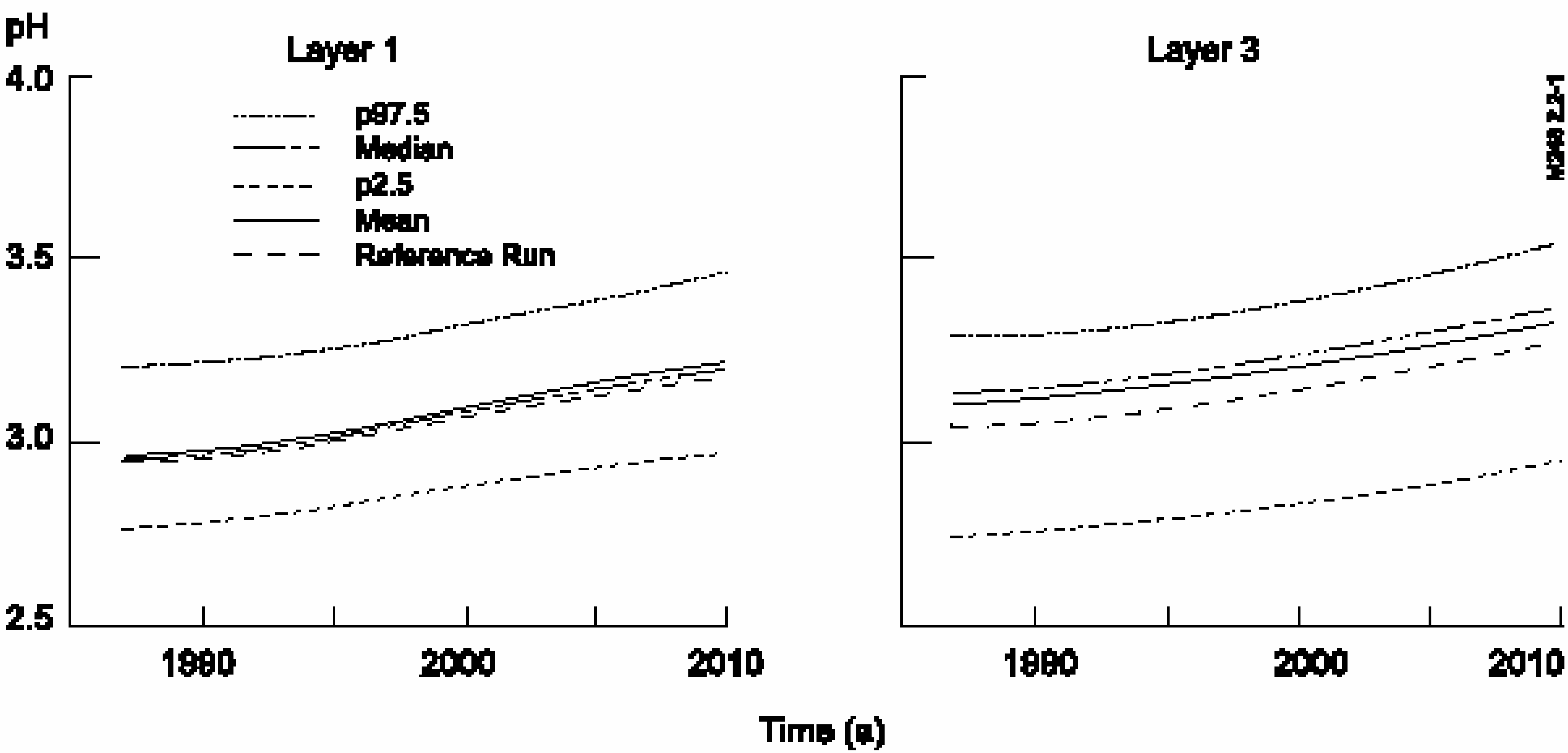


Figure 1 Temporal evolution of the mean, the median, the 97.5 and 2.5 percentiles, and the reference run of the pH in layers 1 and 3.