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Improving the relationship between reference frames by using the Thin-Plate Spline modelling

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Abstract

The adoption of new geodetic reference systems and/or the availability of new reference frames occur, mainly, by the availability of new data sets. For the use of products in different reference frames, methods are necessary to transform the coordinates and modelling the distortions that compromise the transformation. In this paper a method is presented for three-dimensional coordinates transformation with modeling of distortions based on Thin-Plate Splines (TPS) mapping functions. Experiments were performed with real data of SAD 69 (South American Datum – 69) stations in the realization of 1996 (SAD69/96) and its homologous coordinates in SIRGAS2000. In the control points, the values of the root mean square error (RMSE) of the discrepancies in latitude and longitude, respectively, were 0.9 mm and 0.6 mm, and in the check points of 7.8 cm and 6.7 cm. By using distinct points of the network, the comparison of direct and inverse transformation results in the standard deviation were of 0.5 mm in latitude and 0.3 mm in longitude. In the comparison between the TPS model and the ProGriD, the statistical indicators were reduced by 97% (average), indicating that the proposed method is promising.

Keywords: reference systems, coordinate transformation, distortion modelling, thin-plate splines.

1. Introduction

The Brazilin Geodetic System (SGB) changed in 2005, in which the SIRGAS2000 (Geocentric Reference System for the Americas) was adopted as the official Brazilian reference system. The change of geodetic reference system demands studies to promote the support to the users, especially to the conversion of products from the old reference systems for the new one. So, the coordinates transformation and the distortion modeling between referential systems started to be a focus of studies and a latent preoccupation. In 2008 the Brazilian Institute of Geography and Statistics (IBGE) made the computational program called ProGriD available as an effort of promoting to the community of spatial data users a tool that makes easy the transition for the SIRGAS2000 (IBGE, 2009).

The methodology proposed in this paper is based on the modeling with Thin-Plate Spline (TPS) mapping functions, used to determine relative distortions in two sets of corresponding control points. The original formulation TPS was changed for the use of geodetic Cartesian three-dimensional coordinates. The method was checked in experiments with real stations data from the SGB.

2. Thin-Plate Splines

The TPS concept is based on the minimization of the bending energy of a thin metal plate fixed by some control points, as can be seen in Bookstein (1989). This formulation guarantees restrictions on the obtained surface, as for example that the surface is smooth and presents minimum bending energy. According to Bookstein (1989), for a thin metal plate, subject to a smooth curvature, the spline function f(x,y) that minimizes the bending energy is given by the following equation:

$$f(x,y) = a + bx + cy + \sum w_i U(r), \quad (1)$$

where:
a, b, c and w_i (i = 1, ..., n) – coefficients by the function;
U(r) – radial base function; and
r – distance from de point (x, y) to the control point i.

The TPS function is composed by linear combination of radial functions (U (r)), where the values of the function are obtained by the differences (or distances) between the coordinates of the control points (used in the model computation).One of the TPS applications is on the generation of R^2 to R^2 maps, relating two sets of homologous points. In great part of the solutions using TPS, surfaces of interpolation f(x, y) are produced in function of two-dimensional coordinates (x, y). Details about the mapping functions in two dimensions can be found in Bookstein (1989). The expansion of the model for the three-dimensional case is indicated by Bookstein (1989) and the principal changes happen in the radial base function.

The base of the analyses of TPS is the radial base function U. The function U change depends on the space dimension related to the treated problem. In the two-dimensional space (R^2) the function U is given by the Equation (2) and in the three-dimensional case (R^3), U is given by the Equation (3) (BOOKSTEIN, 1989).

$$z(x,y) = U(r) = -r^2.\log\left(r^2\right), \quad (2)$$

$$z(x,y) = U(r) = |\, r\,|, \quad (3)$$

where r is the distance to the origin of cartesian system, that is defined by the coordinates of each control point.

From the radial base function and the Equation (1) it is possible to define the TPS mapping functions in X, Y and Z, given by the equations:

$$\begin{aligned} X_2 &= a_0 + a_1X_1 + a_2Y_1 + a_3Z_1 + \textstyle\sum_{i=1}^{n} u_iU(r_i), \\ Y_2 &= a_4 + a_5X_1 + a_6Y_1 + a_7Z_1 + \textstyle\sum_{i=1}^{n} v_iU(r_i), \\ Z_2 &= a_8 + a_9X_1 + a_{10}Y_1 + a_{11}Z_1 + \textstyle\sum_{i=1}^{n} w_iU(r_i), \end{aligned} \quad (4)$$

where:

X_2, Y_2, Z_2 - geodetic cartesian coordinate in the reference system of destiny;
X_1, Y_1, Z_1 - geodetic cartesian coordinate in the reference system of origin;
$a_0, a_1, ..., a_{11}, u_1, ..., u_n, v_1, ..., v_n, w_1, ..., w_n$ - TPS coefficients in *X*, *Y* and *Z*;
$U(r_i)$ - application of the Equation (3) by using as argument the Euclidian distance in three-dimensional space (r_i) from a control point to the others.

To determinate the coefficients of the TPS model is necessary the resolution of the system of equations composed by the Equations (4) and the constraints given by the following equations:

$$\begin{aligned}
&\textstyle\sum_{i=1}^{n} u_i = 0, && \textstyle\sum_{i=1}^{n} v_i = 0, && \textstyle\sum_{i=1}^{n} w_i = 0, \\
&\textstyle\sum_{i=1}^{n} u_i X_i = 0, && \textstyle\sum_{i=1}^{n} u_i Y_i = 0, && \textstyle\sum_{i=1}^{n} u_i Z_i = 0, \\
&\textstyle\sum_{i=1}^{n} v_i X_i = 0, && \textstyle\sum_{i=1}^{n} v_i Y_i = 0, && \textstyle\sum_{i=1}^{n} v_i Z_i = 0, \\
&\textstyle\sum_{i=1}^{n} w_i X_i = 0, && \textstyle\sum_{i=1}^{n} w_i Y_i = 0, && \textstyle\sum_{i=1}^{n} w_i Z_i = 0.
\end{aligned} \tag{5}$$

In the specific case of the SGB network, the stations are distributed irregularly in a territory of continental dimensions. In this case, a problem that can occur in the TPS modeling is that two very near points can have relatively small distances relate to the other points. It implicates that some lines and/or columns of the matrix A (matrix of the coefficients) can be linear combinations of others, causing problems of ill conditioning and, consequently, in the matrix inversion.

Aiming to improve the conditioning of the matrix A, a threshold was adopted for the distance of near points, assuming that for close points of the network the deformations are similar. A pre-processing is carried out based on available control points to identify pair of points in which the distances are smaller (or equal) to the established threshold. In this case, one of these points of the pair is eliminated. Experiments were carried out to establish the threshold of distance, as detailed in Magna Junior (2012). Considering the stations of the SGB, the threshold of 1 km was the one that better attended to the analysed criteria.

3. Proposed Method for Coordinates Transformation

The proposed approach is based on the use of geodetic cartesian coordinates (X, Y and Z) for the TPS modeling. The first phase of the method was the expansion of the TPS model to the three-dimensional space, that is, using coordinates in the 3D space, and the resolution of the system of equations (to estimate the model coefficients). The 3D TPS model, as introduced in the previous section, consists in the solution of a system of linear equations formed by the Equations (4) and (5).

As there is no redundancy in the solution of the system and the matrix A is squared, the vector of the coefficients X can be found by the multiplication of the inverse of the matrix of the coefficients (A^{-1}) and the vector of the observations (L), in the form $X = A^{-1}L$. The individual solution of the coefficients for each component of the coordinates consists in the solution of a system of n + 4 equations by n + 4 unknowns and the vector X is formed for n + 4 coefficients.

The ill conditioning problem in the solution of the TPS coefficients, due the geometry of the SGB stations, were solved by the application of the distance threshold to eliminate some points, as mentioned, followed by the inversion of the matrix A using SVD – Singular Value Decomposition. The SVD approach is a widely used method in analysis and solution of equations system with singular or almost singular matrix (ill conditioned). Details of the method are presented in Press et al., (1992).

The final stage consists in applying the coefficients of the model in points on the realization of origin to determine its coordinates on the destiny realization.

It is important to note that, in function of the magnitude of the geodetic cartesian coordinates, the processing was carried out by using normalized coordinates in the interval [-10:10], intending to avoid numerical problems. At the end of the modeling process, the reverse normalization is carried out to obtain the geodetic cartesian coordinates in the original scale and magnitude.

4. Experiments and Results

The validation of the proposed method was performed by experiments with real data provided by the IBGE, corresponding to the SGB stations, in the two realizations: SAD69/96 and SIRGAS2000.

4.1. Evaluation of the TPS Model with the Total Set of Points

In this experiment there were determined the coefficients of the TPS model by using the total available set of points (4352) taken as control points. The estimated coefficients were applied at the same control points with the purpose of checking the consistency between the functional model and the data set. The analyses were based on the discrepancies computed between the known coordinates (adjusted) and the computed coordinates in the destiny reference system. The same experiment was performed with the ProGriD, aiming to compare both models. Some statistics of the computed discrepancies in the control points are presented in the Table 1.

Table 1: Statistics from the modeling with the totality of the points.

Discrepancies	TPS Model		ProGriD	
	Lat (m)	Lon (m)	Lat (m)	Lon (m)
Average	0.0000	0.0000	-0.0068	-0.0061
Standard Deviation	0.0009	0.0006	0.0474	0.0420
RMSE	0.0009	0.0006	0.0479	0.0424
90% ≤	0.0012	0.0009	0.0432	0.0448

According to the Table 1, when used the TPS model the average discrepancy was null in both latitude and longitude components. The values of the RMSE were of 0.9 mm in latitude and 0.6 mm in longitude. In general, the values near to zero for the statistical indicators suggest that the TPS modeling agree to the desirable characteristic for a model, that is, guarantee the fidelity of the values in the control points. Comparing the TPS versus ProGriD results, a reduction up 97% occurred in all the statistical indicators when used the TPS model. The values of RMSE obtained with the ProGriD were of 47.9 mm in latitude and 42.4 mm in longitude, while in the TPS modeling the values were of 0.9 mm in latitude and 0.6 mm in longitude.

4.2 Evaluation of the Direct and Inverse Transformations in Points non Coincident with the SGB Stations

In this experiment were used 368 points arranged in 2° x 2° grid in latitude and longitude as check points. These points have different coordinates from the SGB stations and were assumed as having its coordinates associated to the SAD69/96. The conversion to the SIRGAS2000 was performed by applying the TPS model obtained with the total set of points. The resultant coordinates of the transformation were converted again to SAD69/96, by using the inverse TPS model. As the initial coordinates were free of errors, due the way as they were defined (mathematically), the differences between the original coordinates and the transformed ones represent the errors from the transformations themselves. The Table 2 shows the results of the experiment with the TPS model and the ProGriD.

Table 2: Statistics of the discrepancies in the direct and inverse transformations.

Discrepancies	TPS Model		ProGriD	
	Lat (m)	Lon (m)	Lat (m)	Lon (m)
Average	0.0000	0.0000	-0.0002	-0.0002
Standard Deviation	0.0005	0.0003	0.0005	0.0004
90% ≤	0.0005	0.0003	0.0011	0.0010

The other values obtained are of around one tenth of millimeter, indicating that the errors, due the transformation by the TPS model, are of small magnitude. In the comparison of the results from TPS versus ProGriD it is possible to verify that both models provided discrepancies of small magnitude (<3.5 mm).

4.3 Evaluation of the Modeling Quality in the Check Points

In this experiment the control points were used to estimate the coefficients of the TPS model and the check points, different from the control points, were used to perform the quality control of the transformation. The total set of data available in the realization SAD69/96 is composed by 4.474 points (by using classic geodetic techniques), where 4.067 are control points and 407 are check points. There were discarded 99 points by the threshold of distance. In the Table 3 some statistical information of the discrepancies in the check points before and after the TPS modeling are shown.

Table 3: Discrepancies in the check points before and after the TPS modeling.

Discrepancies	Before TPS Modeling		After TPS Modeling	
	Lat (m)	Lon (m)	Lat (m)	Lon (m)
Average	-49.9696	-45.7392	0.0080	0.0012
Standard Deviation	3.9509	4.5453	0.0779	0.0676
RMSE	53.7011	46.8038	0.0782	0.0675
90% ≤	54.5998	50.5703	0.0256	0.0067

The magnitude of the discrepancies obtained before the modeling is around 50 m in the latitude and 46 m in the longitude, with respective standard deviation of 3.9 m and 4.5 m. After the TPS modeling, the average discrepancies were reduced to 8.0 mm and 1.2 mm, with the standard deviation of around 78 mm and 68 mm, respectively in latitude and longitude. The values of RMSE were 78.2 mm in the latitude and 67.5 mm in the longitude.

It is possible to verify in the Table 3 that the magnitude of the RMSE is influenced by the discrepancies of few check points located in the most complex regions of modeling, where the discrepancies are relatively superior to the others points (MAGNA JUNIOR, 2012). This can be proved by the analyses of the 90% of the points, where the discrepancies are down to 25.6 mm in latitude and 6.7 mm in longitude.

5. Final Considerations and Conclusions

The present paper aims to collaborate with the process of reference systems transformation, proposing a methodology based on Thin-Plate Splines. The proposed method performs the transformation in the three-dimensional space, aggregating the distortions modeling inherent in the realizations.

The results showed that the proposed method provides coherent results with the mathematical model, with null average discrepancies in the control points. In the direct and inverse transformations the results showed that for the set of points available, the maximum error obtained by the TPS modeling is around 3.5 mm in latitude and 2.6 mm in longitude. In the check points there was a reduction up to 99% of the distortions in each component after the TPS modeling. The comparison between the TPS model and the ProGriD showed that the proposed method provides discrepancies with lower values in the control points after the coordinates transformation and all the statistical indicators considered were reduced up to 97%, when TPS model was considered.

Based on the presented results, it is possible to consider that the TPS modeling is efficient in the transformation of coordinates and in the TPS modeling between realizations of geodetic reference systems, providing a quality transformation and that guarantees integrity to the transformed data. The modeling by TPS is a promising method that can also include the altimetric information, when this information is available, without any adaptation, since it uses geodetic cartesian coordinates.

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